Harold A. Wheeler's Antenna Design Legacy

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Abstract— Harold A. Wheeler had a distinguished career in the field of radio-electronics. He had a unique talent for reducing complex scientific principles to simple forms that were universally helpful to theoreticians and practitioners. Of his many contributions, those related to antenna design are the subject of this paper. The three principal antenna topics in Dr. Wheeler's experience were impedance matching to a transmission line, electrically small antennas, and planar arrays. This paper concentrates on the first two topics and presents two examples of how he developed simple forms that were very helpful and useful to the antenna community. His solutions, although simple in form, were in exact agreement with those based on more rigorous theory.

Index Terms—antenna Q, impedance matching, small antennas

I. Introduction

Harold A. Wheeler (1903 – 1996) had a distinguished career in the field of radio-electronics. He was a pioneer in the design of radio receivers for sound, FM and TV. His invention of automatic volume control (AVC) in 1925 was a major contribution to early radio, and continues today to be a key component in many electronic circuits. In the early days of television he published several significant papers and in 1935 he received the IRE Morris Liebmann Memorial Prize for his paper on TV amplifier problems.

During World War II he was in charge of the design of the SCR-625 mine detector which was used extensively by the Allied forces. He was also a key contributor to the development of the adjunct radar system "identification friend-or-foe" (IFF). At the end of the war he was awarded the Navy Certificate of Commendation.

After the war he started Wheeler Laboratories. He was involved with this group until his retirement in the late 1980s. This group specialized in microwave components, waveguide assemblies, radar and communications, and antennas. In 1964 he received the IEEE's highest award, the Medal of Honor: "For his analyses of the fundamental limitations on the resolution in television systems and on wideband amplifiers, and for his basic contributions to the theory and development of antennas, microwave elements, circuits and receivers."

At the 1984 Centennial Session of the Antennas and Propagation Society Symposium Harold Wheeler presented "Antenna Topics in My Experience" [1] in which he outlined his work in the field of antennas. He presented three topics: wideband matching to a transmission line, small antennas, and planar arrays. His later work on planar arrays is well known and appreciated by the antenna community. This, however, is not the case for his earlier work on electrically small antennas and impedance matching. The author, in recent publications [2-4], has presented Wheeler's work on electrically small antennas and impedance matching in a detailed form, hopefully, to shed more light on Wheeler's contributions in these two areas of antenna design. This paper presents a summary of these publications.

II. Electrically Small Antennas

A. Wheeler and Chu

Typically the names of Wheeler and Chu are associated with electrically small antennas. In many cases there is no distinction made between their contributions. As noted in [2] Wheeler and Chu made individual and distinct contributions to the theory and practice of electrically small antennas. The one common thread was that they both developed a relationship between the Q of an electrically small antenna and an associated physical volume.

Wheeler published the paper "Fundamental Limitations of Small Antennas" in December 1947 [5]. Fig. 1 is the original Fig. 1 from this paper.

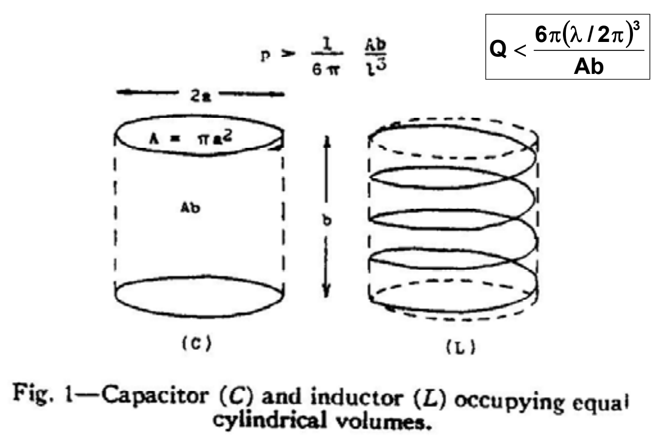

Fig. 1. Wheeler's original figure as it appeared in his 1947 paper. The box insert converts Wheeler's power factor relationship to one involving Q (p = 1/Q).

Wheeler recognized that a small antenna behaved as a lumped-element capacitor or inductor with some radiation resistance. Using simple formulas for these lumped-element components he developed his simple equation for the radiation Q of a small antenna [2]:

$$Q_{Wheeler} = \frac{9}{2}\frac{V_{RS}}{V_E} = \frac{9}{2}\frac{V_{RS}}{kV_{OC}} \tag{1}$$

Where:

V_{RS} = Volume of radiansphere = $\frac{4\pi}{3}\text{Radianlength}^3$

Radianlength = $\frac{\lambda}{2\pi}$

λ = Wavelength

V_E = Effective volume of small antenna = kV_{OC}

V_{OC} = Occupied volume (for a cylindrical antenna it is the volume of the cylinder)

k = Effective volume factor (typically > 1)

Wheeler defined the radianlength and the corresponding radiansphere. The radianlength is a convenient measure for the size of a small antenna. Wheeler defined a small antenna as one whose maximum dimension is less than the radianlength. The radiansphere is the boundary between the near field and the far field of a small antenna.

Wheeler's Equation (1) defines a fundamental limitation for small antennas. The radiation Q of a small antenna is inversely proportional to the physical volume of the antenna. The accuracy of Equation (1) depends on the effective volume factor, which can be accurately determined using quasi-static electromagnetic (lumped-element) theory.

Lan Jen Chu published his paper "Physical Limitations of Omni-Directional Antennas" in December 1948 [6]. His goal was to quantify the super-gain limitations of omni-directional antennas. In the process he derived the relationship for the fundamental theoretical lower bound for the Q of an electrically small antenna. The Chu relationship is [2]:

$$Q_{Chu} = \frac{V_{RS}}{V_{Chu}} \tag{2}$$

Q_{Chu} is the theoretical lower bound for the Q of a small antenna. V_{Chu} is the volume of a sphere whose diameter is the maximum dimension of the small antenna.

After presenting Wheeler's formulas for the capacitor and the inductor antennas, a perspective of the Wheeler and Chu contributions is presented.

B. Wheeler's Formula for the Capacitor Antenna

Wheeler's formula [2] for the capacitor antenna is presented below:

$$Q_{Wheeler(Capacitor)} = 6\pi\frac{\left(\frac{\lambda}{2\pi}\right)^2}{\pi a^2 b}\frac{1}{k_b} = \frac{9}{2}\frac{V_{RS}}{V_E} = \frac{9}{2}\frac{V_{RS}}{V_{OC}}\frac{k_{SC}+\varepsilon_r-1}{{k_{SC}}^2} \tag{3}$$

Where:

a = Disc radius

b = Distance between discs, dipole length

$k_a = k_{SC}k_{FC}$ = Effective volume factor for capacitor antenna

k_{SC} = Shape factor for capacitor antenna

$$k_{SC} = 1 + \frac{4}{\pi}\frac{b}{a}$$

k_{FC} = Fill factor for capacitor antenna

$$k_{FC} = \frac{1}{1+\frac{\varepsilon_r - 1}{k_{SC}}}, \qquad \text{For } \frac{b}{a} < 2$$

ε_r = Relative permittivity of fill (core) material

A computer simulation was used to validate this formula. Fig.2 shows the computer model and the results of the validation. Very close agreement exists between Wheeler's formula and the simulation results.

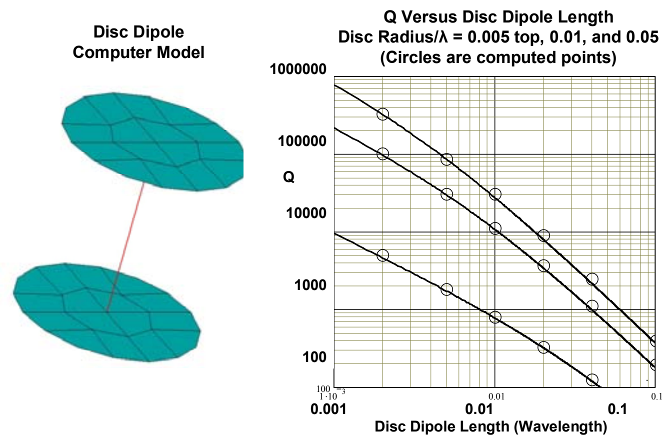

Fig. 2. Validation of Wheeler's formula for the capacitor antenna, $\varepsilon_r = 1$.

C. Wheeler's Formula for the Inductor Antenna

Wheeler's formula [2] for the inductor antenna is presented below:

$$Q_{Wheeler(Inductor)} = 6\pi\frac{\left(\frac{\lambda}{2\pi}\right)^2}{\pi a^2 b}\frac{1}{k_b} = \frac{9}{2}\frac{V_{RS}}{V_E} = \frac{9}{2}\frac{V_{RS}}{V_{OC}}\frac{k_{SL}+1/\mu_r-1}{{k_{SL}}^2} \tag{4}$$

Where:

a = Loop radius

b = Loop axial length

$k_b = k_{SL}k_{FL}$ = Effective volume factor for inductor antenna

k_{SL} = Shape factor for inductor antenna

$k_{SL} \approx 1 + 0.9\frac{a}{b}$, somewhat less than this value if b < a

k_{FL} = Fill factor for inductor antenna

$$k_{FL} \approx \frac{1}{1-\frac{1-1/\mu_r}{k_{SL}}}, \qquad \text{For } \frac{b}{a} > 2$$

μ_r = Relative permeability of fill (core) material

Again, a computer simulation was used to validate this formula. Fig. 3 shows the computer model and the results of the validation. Very close agreement exists between

Wheeler's formula and the simulation results.

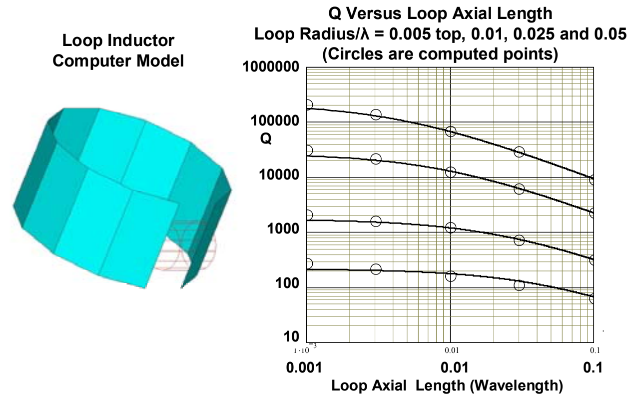

Fig. 3. Validation of Wheeler's formula for the inductor antenna, $\mu_r = 1$.

D. *The Wheeler and Chu Contributions*

The Chu approach to the problem was highly theoretical. He used spherical wave functions and rigorous electromagnetic field theory to determine the lower bound for the Q of a small antenna whose maximum dimension is the diameter of the Chu sphere (volume = V_{Chu}). This antenna can only be realized in theory because it requires that there be no energy storage inside the Chu sphere. Chu's work was remarkable in that he was able to develop a simple lumped-element circuit whose performance was equivalent to that of the small antenna. Equation (2) can be derived using this circuit [2].

Wheeler's approach to the problem was pragmatic. He appreciated the fact that a small antenna behaved essentially as lumped capacitance or inductance. He used relatively simple lumped-circuit analysis to develop his formulas. His formulas are accurate and useful for the design of practical small antennas.

The Wheeler and Chu contributions are substantial. Chu provided the theoretical lower bound for the Q of a small antenna. Wheeler provided accurate formulas for the Q of small antennas. Wheeler's work is useful and helpful to both theoreticians and practitioners. Wheeler, using his formulas, was able to describe one implementation of a small inductor antenna that approached the Chu limit [2].

III. IMPEDANCE MATCHING

A. *Wheeler and Fano*

Wheeler recognized in the 1930s [1] that an antenna behaved as a circuit element and that impedance matching of an antenna to a transmission line was needed in some applications. During the 1940s he developed the art of impedance matching using the reflection chart [7] as the primary tool. Fig. 4 presents the basic circuit diagram for multiple tuning of an antenna [3]. Contiguous tuning circuits alternate between series and parallel tuned circuit. All the tuned circuits are resonant at the same frequency.

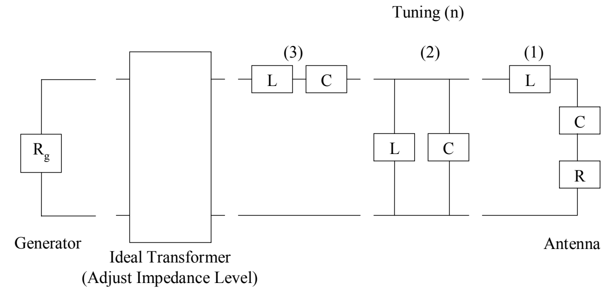

Fig. 4. Circuit diagram for multiple-tuning impedance matching network

Wheeler, using the reflection chart, developed explicit formulas that relate the antenna Q-Bandwidth product to the maximum allowed reflection magnitude for the single-tuned and double-tuned circuits [4]. Wheeler's equations for single and double tuning are:

$$QB = \frac{2\Gamma^{\frac{1}{n}}}{1-\Gamma^{\frac{2}{n}}}, \text{ single tuning } n = 1, \text{ double tuning } n = 2 \quad (5)$$

Where:

$$B = \text{Bandwidth Ratio} = \frac{f_H - f_L}{\sqrt{f_H f_L}} = \frac{f_H - f_L}{f_0}$$

f_H = Highest frequency
f_L = Lowest Frequency
f_0 = Resonant frequency
Γ = Maximum allowed reflection magnitude

Fano [8, 2], using rigorous linear circuit theory, developed a complete set of equations for all levels of tuning ranging from n = 1 to n = ∞. His equations are:

$$QB = \frac{2\sin\left(\frac{\pi}{2n}\right)}{\sinh(a)-\sinh(b)}$$
$$\frac{\tanh(na)}{\cosh(na)} = \frac{\tanh(nb)}{\cosh(b)} \quad (6)$$
$$\frac{\cosh(nb)}{\cosh(na)} = \Gamma$$

The relationship between the QB product and Γ is a set of transcendental simultaneous equations. It is remarkable that Wheeler's formulas for single and double tuning are in exact agreement with Fano's equations, since the forms of the relationship are so different.

B. *Wheeler's Single Tuning Equation*

The block diagram for the single-tuned circuit is shown in Fig. 5. Fig. 6 is a reflection chart showing Wheeler's optimum solution for the single-tuned network.

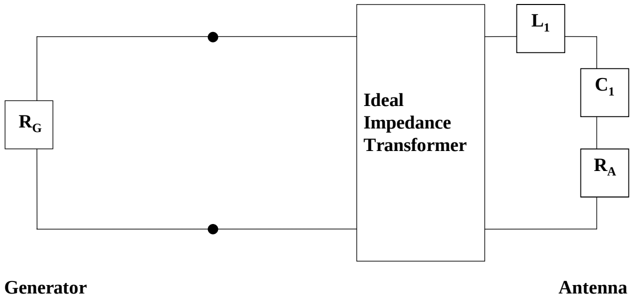

Fig. 5. Diagram for single-tuned impedance matching network

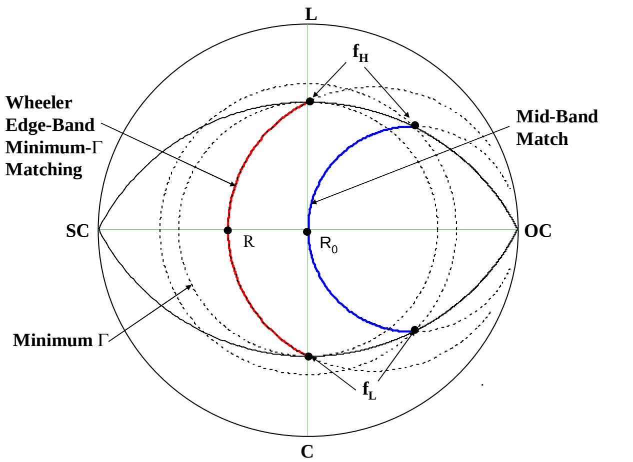

Fig. 6. Reflection chart showing optimum single-tuning impedance matching

Wheeler starts with, what he referred to, as the "mid-band match" condition. For this case the network is adjusted so that the tuning and the impedance transformation results in impedance match (zero reflection magnitude) being achieved at the resonant frequency. Clearly, this is not the optimum single-tuned design. Wheeler indicates that if the impedance transformation is adjusted such the edge-band frequencies lie on the vertical axis of the chart then, the optimum single-tuned impedance matching (minimum maximum reflection magnitude) is achieved. The proof lies in simple geometrical considerations.

The relevant equations for the derivation of his single-tuned formula are presented below:

$$Z(f_H) = R + j\omega_0 L\left(\frac{f_H}{f_0} - \frac{f_0}{f_H}\right) = R\left(1 + \frac{j\omega_0 L}{R}\left(\frac{f_H}{f_0} - \frac{f_L}{f_0}\right)\right)$$

$$z(f_H) = \frac{Z(f_H)}{R_0} = \frac{R}{R_0}(1 + jQB) = \exp(i\phi) = \cos(\phi) + j\sin(\phi)$$

$$\left|\frac{R}{R_0}(1 + jQB)\right| = 1 \quad \frac{R_0}{R} = \sqrt{1 + (QB)^2} \quad \tan(\phi) = QB$$

$$\Gamma = \left|\frac{z(f_H) - 1}{z(f_H) + 1}\right| = \sqrt{\frac{(\cos(\phi) - 1)^2 + \sin(\phi)^2}{(\cos(\phi) + 1)^2 + \sin(\phi)^2}} = \tan(\phi/2)$$

$$\tan(\phi) = \frac{2\tan(\phi/2)}{1 - \tan(\phi/2)^2}$$

$$QB = \frac{2\Gamma}{1 - \Gamma^2} = \frac{2\Gamma^{\frac{1}{n}}}{1 - \Gamma^{\frac{2}{n}}} \quad n = 1 \tag{7}$$

C. Wheeler's Double Tuning Equation

The block diagram for the double-tuned circuit is shown Fig. 7. Fig. 8 (a), (b), and (c) are reflection charts showing Wheeler's optimum solution for the double-tuned network.

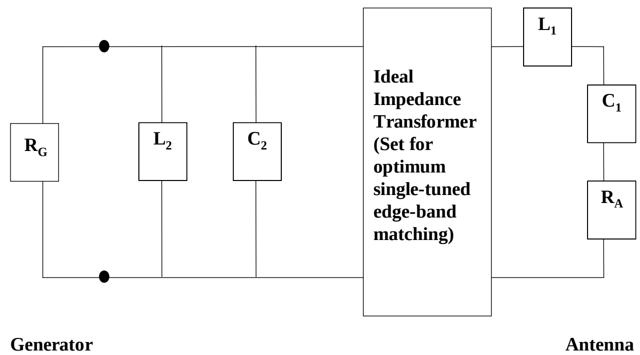

Fig. 7. Diagram for double-tuned impedance matching network

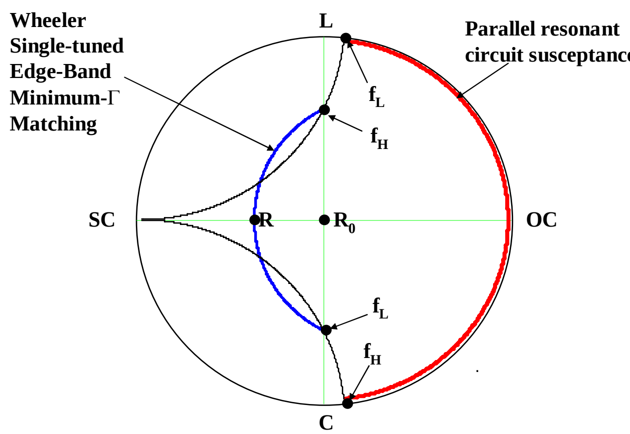

Fig. 8 (a). Reflection chart showing optimum double-tuning impedance matching

The development of the optimum double tuning starts with the optimum single-tuned case. As shown in Fig. 8 (a), the susceptance of the parallel resonant circuit is set equal in magnitude and opposite in sign to the susceptance of the single-tuned circuit at the edge-band frequencies. Fig. 8 (b) shows the resulting locus of the double-tuned network. Simple geometry (similar triangles) is used to show that $\Gamma_2 = \Gamma_1^{\ 2}$. Fig. 8 (c) shows the classic optimum double-tuned locus with the frequency band extended beyond the operating band.

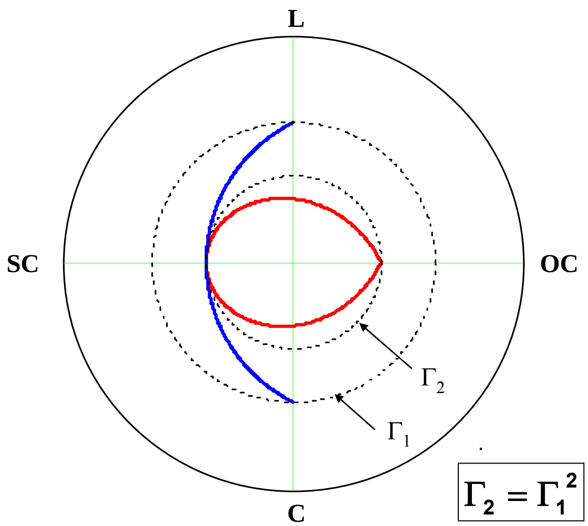

Fig. 8 (b). Reflection chart showing optimum double-tuning impedance matching

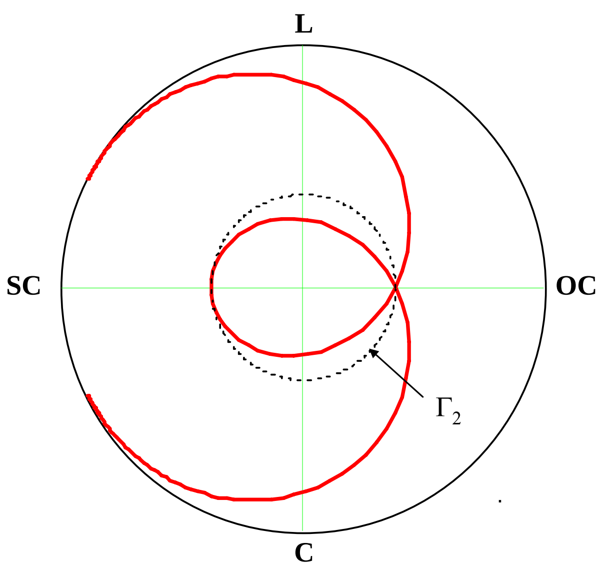

Fig. 8 (c). Reflection chart showing optimum double-tuning impedance matching

Wheeler's formula for double tuning is derived directly from the single-tuned formula.

$$QB = \frac{2\Gamma_1}{1-\Gamma_1^{\,2}}$$

$$\Gamma_1 = \sqrt{\Gamma_2}$$

$$QB = \frac{2\sqrt{\Gamma_2}}{1-\Gamma_2} = \frac{2\Gamma^{\frac{1}{n}}}{1-\Gamma^{\frac{2}{n}}}, \quad n = 2 \tag{8}$$

Where:

Γ_1 = Maximum single-tuned reflection magnitude

Γ_2 = Maximum double-tuned reflection magnitude

C. The Wheeler and Fano Contributions

The contrast between the Wheeler approach and the Fano approach is interesting. Fano, for the most part, relied heavily on mathematical rigor; Wheeler, on the other hand, reduced the problem to a form where the solution was apparent by simple geometrical considerations.

Fano's approach was comprehensive. It provided a complete picture of the basic limitations for the impedance matching of arbitrary impedances. In retrospect, the community benefits from both the Wheeler and Fano contributions.

E. Summary of Impedance Matching Formulas

A summary of the impedance matching circuits described above is presented in the table below [4]:

Impedance Matching Circuit	Equation	QB for V = VSWR = 2
Single-tuned mid-band match (Non Fano)	$QB = \frac{2\Gamma}{\sqrt{1-\Gamma^2}} = \frac{V-1}{\sqrt{V}}$	QB = 0.707
Single-tuned edge-band matching (Wheeler-Fano, n = 1)	$QB = \frac{1}{\sinh\left(\ln\left(\frac{1}{\Gamma}\right)\right)} = \frac{2\Gamma}{1-\Gamma^2} = \frac{V^2-1}{2V}$	QB = 0.750
Double-tuned matching (Wheeler-Fano, n = 2)	$QB = \frac{1}{\sinh\left(\frac{1}{2}\ln\left(\frac{1}{\Gamma}\right)\right)} = \frac{2\sqrt{\Gamma}}{1-\Gamma} = \sqrt{V^2-1}$	QB = 1.732
Infinite-tuned matching (Fano-Bode, n = ∞)	$QB = \frac{\pi}{\ln\left(\frac{1}{\Gamma}\right)} = \frac{\pi}{\ln\left(\frac{V+1}{V-1}\right)}$	QB = 2.860

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In 1958 he received the BEE from Manhattan College, and in 1963 he received the MSEE from the Polytechnic Institute of Brooklyn. He has published more than 30 papers in IEEE and Institute of Navigation publications. He has been awarded 46 U.S. patents and has received several significant awards from the IEEE and BAE Systems. He received two Wheeler awards, the IEEE Antennas and Propagation Society Harold A. Wheeler Best Applications Prize Paper Award in 1987, and the IEEE Long Island Section Harold A. Wheeler Award in 1993.