Rebuttal to "Fano Limits on Matching Bandwidth"

Alfred R. Lopez

ARL Associates
4 Sarina Drive, Commack, NY 11725-1806
Tel: +1 (631) 499-2987; Fax: +1 (631) 462-0320; E-mail: al.lopez@ieee.org

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Hansen [1] implied that Lopez [2] had an error in his article, "Review of Narrowband Impedance-Matching Limitations." Hansen writes:

> Coefficients a and b, multiplied by n, are given by Lopez [2] in Table 2. These do not match the exact values of my Table 1 or Table 2. Lopez does not state what value of VSWR his coefficients are for.

Lopez's coefficients a_n are not equal to the Fano [3] "a" multiplied by n (the number of tuning circuits), as stated by Hansen ($a_n \neq an$). The coefficients $a_1 = 1$ and $a_2 = 2$, for single and double tuning, were first derived by Lopez [4] completely independently of Fano's work. These coefficients are used in a simple equation relating Q, the bandwidth ratio, $B = \left(f_{High} - f_{Low}\right)/\sqrt{f_{High} f_{Low}}$, and the maximum reflection magnitude, Γ:

$$QB_1(\Gamma) = \frac{1}{\sinh\left[\frac{1}{a_1}\ln\left(\frac{1}{\Gamma}\right)\right]} = \frac{1}{\sinh\left[\ln\left(\frac{1}{\Gamma}\right)\right]},$$
$$QB_2(\Gamma) = \frac{1}{\sinh\left[\frac{1}{a_2}\ln\left(\frac{1}{\Gamma}\right)\right]} = \frac{1}{\sinh\left[\frac{1}{2}\ln\left(\frac{1}{\Gamma}\right)\right]}. \quad (1)$$

The coefficients a_1 and a_2 are not a function of Γ or of the VSWR.

Lopez [4] went on to develop an approximate formula for the Q-bandwidth product for the case where $\Gamma > 1/3$ for all values of n:

$$QB_n(\Gamma) \approx \frac{a_n}{\ln\left(\frac{1}{\Gamma}\right)}. \quad (2)$$

Lopez [2], using *MATHCAD* to solve Fano's equations, revised the coefficients, a_n, and determined the coefficients b_n for a very accurate equation relating the Q-bandwidth product to Γ for *all* values of Γ and for $n = 1$ to 8 and $n = \infty$:

$$QB_n(\Gamma) = \frac{1}{b_n \sinh\left[\frac{1}{a_n}\ln\left(\frac{1}{\Gamma}\right)\right] + \frac{1-b_n}{a_n}\ln\left(\frac{1}{\Gamma}\right)} \quad (3)$$

The Lopez coefficients a_n, b_n have no dependence on Γ.

Hansen's [1] Table 3, "Maximum bandwidth improvement factors," $MBIF_m$, can be simply reproduced using Equation (3). The additional number of matching circuits beyond the single tuned circuit is equal to m. Note that $m+1=n$.

$$MBIF_m = \frac{\sinh\left[\ln\left(\frac{1}{\Gamma}\right)\right]}{b_{m+1}\sinh\left[\frac{1}{a_{m+1}}\ln\left(\frac{1}{\Gamma}\right)\right] + \frac{1-b_{m+1}}{a_{m+1}}\ln\left(\frac{1}{\Gamma}\right)}. \quad (4)$$

Hansen's Table 3 is reproduced here as Table 1, including the Lopez values. The Lopez values were rounded off to two decimal places for $m = 2$, 3, and 4. This is consistent with the accuracy of his computed values for a_n and b_n.

From Table 1, and for the case of VSWR = 2, it can be determined that the percentage bandwidth increase for double tuning over single tuning is 131%, and that it is 24% for triple tuning over double tuning. These are the same values presented by Lopez [5] in his Figure 1.

How Do Hansen [1] and Lopez [2] Differ?

Hansen solved the Fano equations for $n = 1$ to 5 and for $n = \infty$, and for VSWR = 2 and VSWR = 5.828. He quantified the law of diminishing returns for the number of tuning circuits greater than two. Lopez also quantified the same law for *all* values of VSWR. The Hansen and Lopez results were the same for the VSWR = 2 and VSWR = 5.828 cases.

What Lopez did was unique. He developed (over a span of 30 years) the relatively simple equation that is equivalent the Fano simultaneous equations. Table 2 presents the Fano and the Lopez-Fano equations for $n = 1$, 2, and 3.

Hansen and Lopez agreed on one thing: the law of diminishing returns for the number of tuning circuits greater than two.

Table 1. Maximum bandwidth improvement factors.

Number of Additional Matching Circuits			VSWR = 2 ($\Gamma = 1/3$)		Half-Power ($\Gamma = 1/\sqrt{2}$)	
m	a_{m+1}	b_{m+1}	**Hansen**	**Lopez Equation (4)**	**Hansen**	**Lopez Equation (4)**
1	2	1	2.3094	2.3094	2.0301	2.0301
2	2.413	0.678	2.8596	2.86	2.4563	2.46
3	2.628	0.474	3.1435	3.15	2.6772	2.68
4	2.755	0.347	3.3115	3.31	2.8083	2.81
∞	π	0	3.8128	3.8128	3.2049	3.2049

Table 2. The Fano and Lopez-Fano equations.

n	Fano	Lopez-Fano
1	$QB(\Gamma) = \frac{2\sin(\pi/2)}{\sinh[a(\Gamma)] - \sinh[b(\Gamma)]}$ $\frac{\tanh[a(\Gamma)]}{\cosh[a(\Gamma)]} = \frac{\tanh[b(\Gamma)]}{\cosh[b(\Gamma)]}$ $\frac{\cosh[b(\Gamma)]}{\cosh[a(\Gamma)]} = \Gamma$	$QB(\Gamma) = \frac{1}{\sinh\left[\ln\left(\frac{1}{\Gamma}\right)\right]}$ January 1973, [2] Equation (5), Table 2
2	$QB(\Gamma) = \frac{2\sin(\pi/4)}{\sinh[a(\Gamma)] - \sinh[b(\Gamma)]}$ $\frac{\tanh[2a(\Gamma)]}{\cosh[a(\Gamma)]} = \frac{\tanh[2b(\Gamma)]}{\cosh[b(\Gamma)]}$ $\frac{\cosh[2b(\Gamma)]}{\cosh[2a(\Gamma)]} = \Gamma$	$QB(\Gamma) = \frac{1}{\sinh\left[\frac{1}{2}\ln\left(\frac{1}{\Gamma}\right)\right]}$ January 1973, [2] Equation (5), Table 2
3	$QB(\Gamma) = \frac{2\sin(\pi/6)}{\sinh[a(\Gamma)] - \sinh[b(\Gamma)]}$ $\frac{\tanh[3a(\Gamma)]}{\cosh[a(\Gamma)]} = \frac{\tanh[3b(\Gamma)]}{\cosh[b(\Gamma)]}$ $\frac{\cosh[3b(\Gamma)]}{\cosh[3a(\Gamma)]} = \Gamma$	$QB(\Gamma) = \frac{1}{b_3 \sinh\left[\frac{1}{a_3}\ln\left(\frac{1}{\Gamma}\right)\right] + \frac{1-b_3}{a_3}\ln\left(\frac{1}{\Gamma}\right)}$ $a_3 = 2.413$, $b_3 = 0.678$ June 2004, [2] Equation (7), Table 2

Hansen's statement, "(Fano) coefficients a and b, multiplied by n, are given by Lopez [2] in Table 2," was not correct. There was no error in the Lopez paper as implied by Hansen.

References

1. R. C. Hansen, "Fano Limits on Matching Bandwidth," *IEEE Antennas and Propagation Magazine*, **47**, 3, June 2005, pp. 89-90.

2. A. R. Lopez, "Review of Narrowband Impedance-Matching Limitations," *IEEE Antennas and Propagation Magazine*, **46**, 4, August 2004, pp. 88-90.

3. R. M. Fano, "Theoretical Limitations of the Broadband Matching of Arbitrary Impedances," *J. Franklin Institute*, **249**, January 1950, pp. 57-83; February, 1950, pp. 139-154.

4. A. R. Lopez, "WL Multiple Tuning Techniques," Hazeltine Wheeler Laboratory Memorandum, G300-73-RL9012, January 1973 (available in electronic format upon request from the author).

5. A. R. Lopez, "More on Narrowband Impedance-Matching Limitations," *IEEE Antennas and Propagation Magazine*, **46**, 6, December 2004, p. 102.

Ideas for Antenna Designer's Notebook

Ideas are needed for future issues of the Antenna Designer's Notebook. Please send your suggestions to Tom Milligan and they will be considered for publication as quickly as possible. Topics can include antenna design tips, equations, nomographs, or shortcuts, as well as ideas to improve or facilitate measurements.