Tom Milligan
Milligan & Associates
8204 West Polk Place
Littleton, CO 80123
(303) 977-7268
(303) 977-8853 (Fax)
TMilligan@ieee.org (e-mail)

Review of Narrowband Impedance-Matching Limitations

Alfred R. Lopez

ARL Associates
4 Sarina Drive, Commack, NY 11725-1806 USA
Tel: +1 (631) 499-2987; Fax: +1 (631) 462-0320; E-mail: al.lopez@ieee.org

Keywords: Impedance matching; Q factor; tuning; multifrequency antennas

Many antennas can be characterized by their radiation Q (the ratio of reactance to radiation resistance). A classic problem is determining the maximum possible bandwidth,

$$B_n = \left(f_{high} - f_{low}\right) / \sqrt{f_{high} f_{low}},$$

constrained by the maximum permissible reflection magnitude, R, and the number of tuned circuits, n, in the impedance-matching circuit. Figure 1 presents a schematic diagram for multiple-tuning networks, for an example antenna characterized by a series combination of capacitance and resistance. The tuned circuits are synchronous. Typically, B_n is less than one-half (narrowband), and the QB_n product is less than three.

Bode [1] determined the relationship between QB_n and R for the case of an infinite number of tuned circuits:

$$QB_\infty = \frac{\pi}{\ln\left(\frac{1}{R}\right)}. \tag{1}$$

Fano [2] determined the relationship between QB_n and R for any number of tuned circuits. His solution was in terms of a set of simultaneous equations:

$$QB_n = \frac{2\sin\left(\frac{\pi}{2n}\right)}{\sinh(a) - \sinh(b)}, \tag{2}$$

$$\frac{\tanh(na)}{\cosh(a)} = \frac{\tanh(nb)}{\cosh(b)}, \tag{3}$$

$$R = \frac{\cosh(nb)}{\cosh(na)}. \tag{4}$$

MATHCAD computer software was used to solve this set of equations. Fano's Figure 19 is reproduced in Figure 2.

Lopez [3] had, as an objective, the quantification of the improvement factor when the impedance-matching circuit is increased by one tuning level. He developed the following equations:

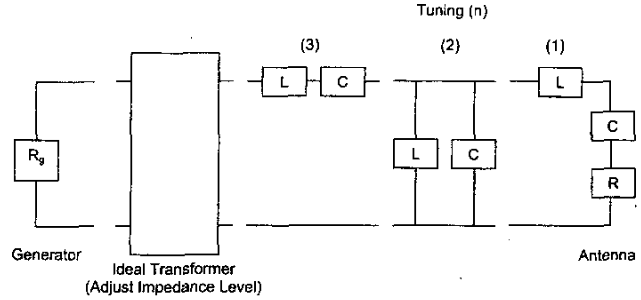

Figure 1. A multiple-tuning impedance-matching network (contiguous tuning elements alternate between series and parallel elements; all tuning elements are synchronous).

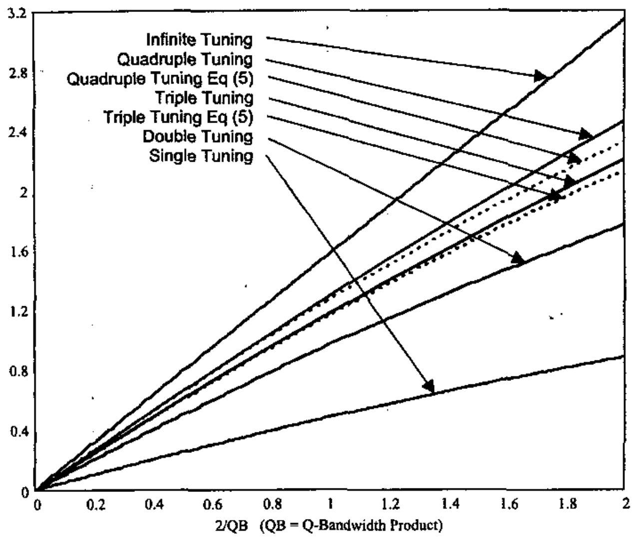

Figure 2. $\ln(1/R)$ **(vertical axis) as a function of** $2/QB$ **(Fano's [2] Figure 19).**

Table 1. The bandwidth increase for increased tuning, $R > 1/3$**,** $B_n/B_{n-1} = a_n/a_{n-1}$**.**

Number of Tuned Circuits (n)	a_n		Percent Bandwidth Increase $[(B_n/B_{n-1})-1]100$
	January 1973	June 2004 (based on Fano)	
1	1	1	
2	2	2	100
3	2.33	2.41	20
4	2.67	2.63	9.1
5	2.76	2.76	4.6
6	2.84	2.84	3.3
7	2.89	2.90	1.8
8	2.93	2.94	1.7
∞	π	π	

Table 2. The coefficients a_n **and** b_n**.**

n	a_n	b_n
1	1	1
2	2	1
3	2.413	0.678
4	2.628	0.474
5	2.755	0.347
6	2.838	0.264
7	2.896	0.209
8	2.937	0.160
∞	π	0

$$QB_n = \frac{1}{\sinh\left[\frac{1}{a_n}\ln\left(\frac{1}{R}\right)\right]}, \tag{5}$$

$$QB_n \approx \frac{a_n}{\ln\left(\frac{1}{R}\right)} \text{ for } R > \frac{1}{3}. \tag{6}$$

Equation (5) is in exact agreement with the Fano relationship for the cases of $n=1$ and $n=2$. Figure 2 shows the difference between the Fano relationship and Equation (5) for the triple- and quadruple-tuning cases. Equation (6) is in exact agreement with the Fano relationship for all values of n as R approaches 1. An initial set of constants, a_n, was defined in [3], and recently revised when the Fano equations were solved using *MATHCAD*. Table 1 presents the old and the new values for a_n, and the bandwidth-increase factor as n increases. It is clear that double tuning provides a substantial increase in bandwidth. It is not clear that a 20% increase in bandwidth for triple tuning will outweigh the added complexity of another level of tuning.

Equation (5) has been revised for improved accuracy:

$$QB_n = \frac{1}{b_n \sinh\left[\frac{1}{a_n}\ln\left(\frac{1}{R}\right)\right] + \frac{(1-b_n)}{a_n}\ln\left(\frac{1}{R}\right)}. \tag{7}$$

The coefficients are presented in Table 2.

In 1951, Tanner [4] interpreted Fano's work, and presented design curves that were more suitable for practical antenna engineers. Jasik [5] published these curves in the first edition of his handbook. Unfortunately, these curves do not include the single- and double-tuning cases. Figure 3 adds these cases to the Tanner design chart, and it also provides improved accuracy for triple and quadruple tuning.

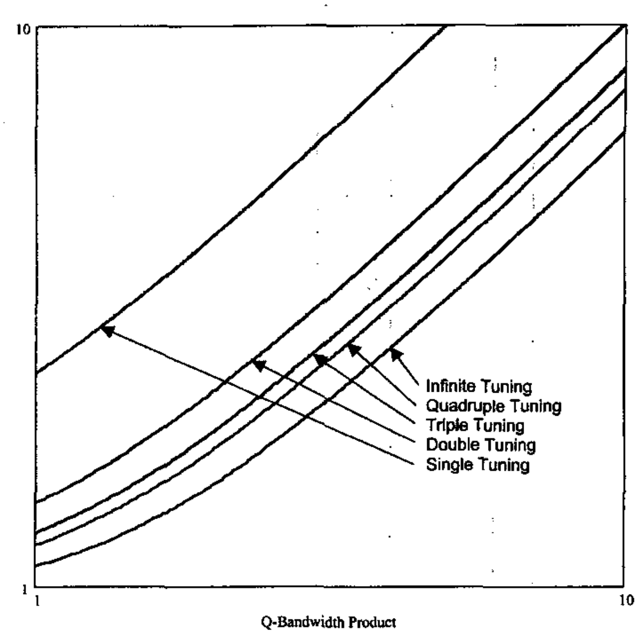

Figure 3. The maximum standing-wave ratio (vertical axis) as a function of QB **(Tanner [4]).**

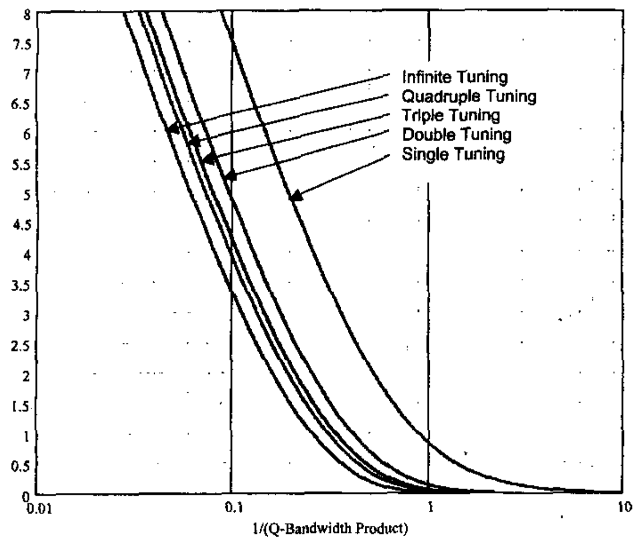

Figure 4. The maximum impedance mismatch loss (in dB, vertical axis) as a function of $1/QB$ (Matthaei, Young, and Jones [6], Figure 4.09-3).

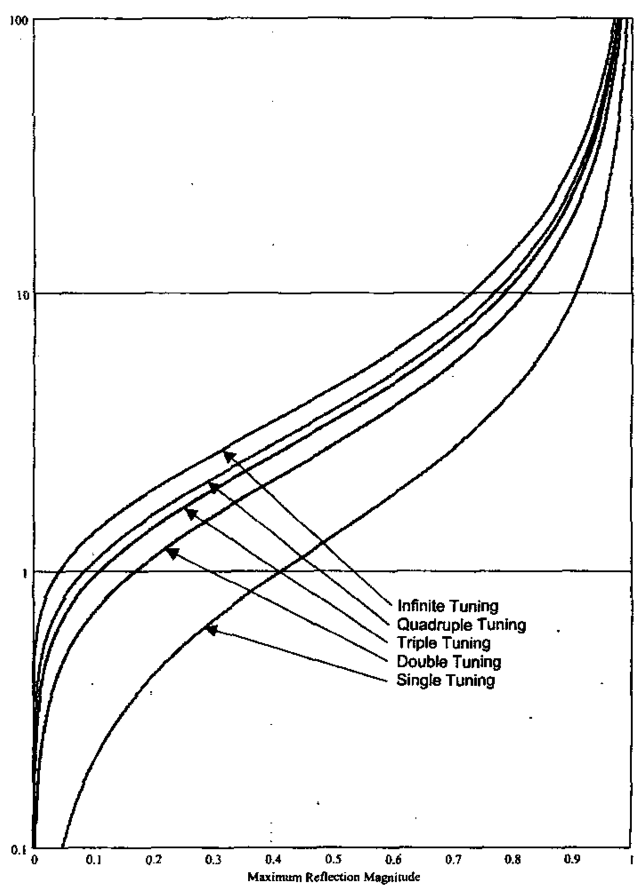

Figure 5. QB (vertical axis, logarithmic scale) as a function of R (Wheeler [7], Figure 3).

Matthaei, Young, and Jones [6] published curves of maximum impedance-mismatch loss in dB versus $1/Q$. They normalized the bandwidth to one. The horizontal scale can be relabeled as $1/QB$. Equation (7) was used to reproduce their results, which are presented in Figure 4.

Wheeler [7] published curves of QB versus R for single, double, and infinite tuning. Figure 5 includes triple and quadruple tuning for this presentation. The scale for the Q-bandwidth product was changed to increase the range.

The derivation of the relationships in [3] was based on the teaching of Harold A. Wheeler [8, 9, 10].

References

1. H. W. Bode, *Network Analysis and Feedback Amplifier Design*, New York, Van Nostrand, 1945, p. 367.

2. R. M. Fano, "Theoretical Limitations on the Broadband Matching of Arbitrary Impedances," Research Lab. Electronics, MIT Tech Report 41, January 1948; also *Journal of the Franklin Institute*, **249**, 1, January, 1950.

3. A. R. Lopez, "WL Multiple Tuning Techniques," Hazeltine Wheeler Laboratory Memorandum, G300-73-RL9012, January 1973 (available in electronic format upon request from the author).

4. R. L. Tanner, "Theoretical Limitations to Impedance Matching," *Electronics*, **24**, February 1951, pp. 234-242.

5. H. Jasik, *Antenna Engineering Handbook*, New York, McGraw-Hill, 1961, pp. 2-46–2-50.

6. G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Norwood, MA, Artech House, 1980 (reprint of edition published by McGraw Hill, 1964).

7. H. A. Wheeler, "Small Antennas," IEEE Transactions on Antennas and Propagation, **AP-23**, 4, July 1975.

8. H. A. Wheeler, "Wideband Impedance Matching," Wheeler Lab Report 418, May 1950.

9. H. A. Wheeler, "Wideband Impedance Matching Between a Resonant Antenna and a Line," Wheeler Lab Report 559, September 1952.

10. H. A. Wheeler, "Impedance Tests of Single or Coupled Resonators," Wheeler Lab Report 665, January 1955.

Ideas for Antenna Designer's Notebook

Ideas are needed for future issues of the Antenna Designer's Notebook. Please send your suggestions to Tom Milligan and they will be considered for publication as quickly as possible. Topics can include antenna design tips, equations, nomographs, or shortcuts, as well as ideas to improve or facilitate measurements.