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HAZELTINE CORPORATION
Wheeler Laboratory

MEMO TO: ANTENNA ENGINEERS

DATE: January 16, 1973

FROM: A. R. Lopez

SERIAL: G300-73-RL9012

SUBJECT: WL MULTIPLE TUNING TECHNIQUES

This memo reviews the WL multiple tuning techniques, it presents equations relating bandwidth (BW), power factor (PF), and minimum reflection circle (minimum circle of radius R which bounds reflection over bandwidth). It gives some rules for the bandwidth increase available from multiple tuning relative to a single-tuned circuit.

Figure 1 shows the equivalent networks for single and double tuning an antenna. The objective is to minimize the maximum reflection or impedance-mismatch loss between the antenna and generator over a desired frequency bandwidth, for a certain degree of complexity in the matching network (single, double or multiple tuning).

Figure 2 presents a summary of the basic single and double tuning concepts. For the case of a given load power factor and bandwidth, it shows the familiar relationship between the minimum reflection circles for single and double tuning ($R_1 = \sqrt{R_2}$). (It should be understood that for the double tuned circuit both primary and secondary circuits are tuned to the same frequency.) Figure 2 also presents a simple derivation of equations relating the bandwidth, power factor and the minimum reflection circle for single and double tuning.

Figure 3 presents a plot of the BW/PF ratio versus reflection tolerance for single and double tuned circuits and the theoretical upper bound for an infinite number of tuned circuits (Ref. 5).

Figure 4 developes the pattern for the BW/PF and the degree of tuning. The result for $R > 1/3$ leads to the following rules for the case of a given load power factor and reflection tolerance.

Number of Tuned Circuits	Bandwidth-Increase Factor Relative to a Single Tuned Circuit for $R > 1/3$
1	1
2	2
3	2 1/3
⋮	⋮
infinite	π

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For the general case of the n-tuned circuit, the development presented in Figure 4 indicates that the actual relationship for BW, PF and R may be

$$\frac{BW}{PF} = \frac{2R^{1/S_n}}{1-R^{2/S_n}} = \frac{1}{\sinh\left(\frac{1}{S_n}\ \ln\frac{1}{R}\right)}$$

where

$$S_n = \sum_{m=1}^{n} a_m$$

and a_m is the mth term of the series (Jolley, "Summation of Series", Dover, (410) p. 76; 1961)

$$1 + 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{5}\left(\frac{2}{3}\right)^2 + \frac{1}{5}\left(\frac{2}{3}\right)^2 + \frac{1}{7}\left(\frac{2\cdot 4}{3\cdot 5}\right)^2 + \frac{1}{7}\left(\frac{2\cdot 4}{3\cdot 5}\right)^2 + \ldots\ldots = \pi$$

This equation is correct for $n = 1$ and 2 for all values of R and BW/PF; for the case of $R > 1/3$ it is in agreement (within plotting accuracy) with the results given in Refs. 5, 6, 8. (These references do not plot the case of small reflection tolerance and small BW/PF.) It is also noted that the equation does not violate the theoretical upper bound for any R and for any n.

$$\frac{BW}{PF} \leq \frac{\pi}{\ln\ 1/R}$$

REFERENCES:

1. H. A. Wheeler, "Wideband Impedance Matching", WL Report 418; May, 1950.

2. H. A. Wheeler, "Wideband Impedance Matching Between a Resonant Antenna and a Line", WL Report 559; Sept., 1952.

3. H. A. Wheeler, "Impedance Tests of Single or Coupled Resonators", WL Report 665; Jan., 1955.

4. H. A. Wheeler "Handbook of Electromagnetics", pp. 12.04.01-.05; 1968.

5. R. M. Fano, "Theoretical Limitations on the Broadband Matching of Arbitrary Impedances", Research Lab. Electronics, -MIT, TECH Report 41; Jan., 1948.

6. R. L. Tanner, "Theoretical Limitations to Impedance Matching", Electronics, vol. 24, pp. 234-242, Feb. 1951.

7. A. Vassiliadis, R. L. Tanner, "Evaluating the Impedance Broadbanding Potential of Antennas", IRE Trans., vol. AP-6, pp. 226-231, July 1958.

8. H. Jasik, "Antenna Engineering Handbook", McGraw-Hill, New York, pp. 2-46 to 2-50; 1961.

A. R. Lopez

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Attachments: Figures 1-4

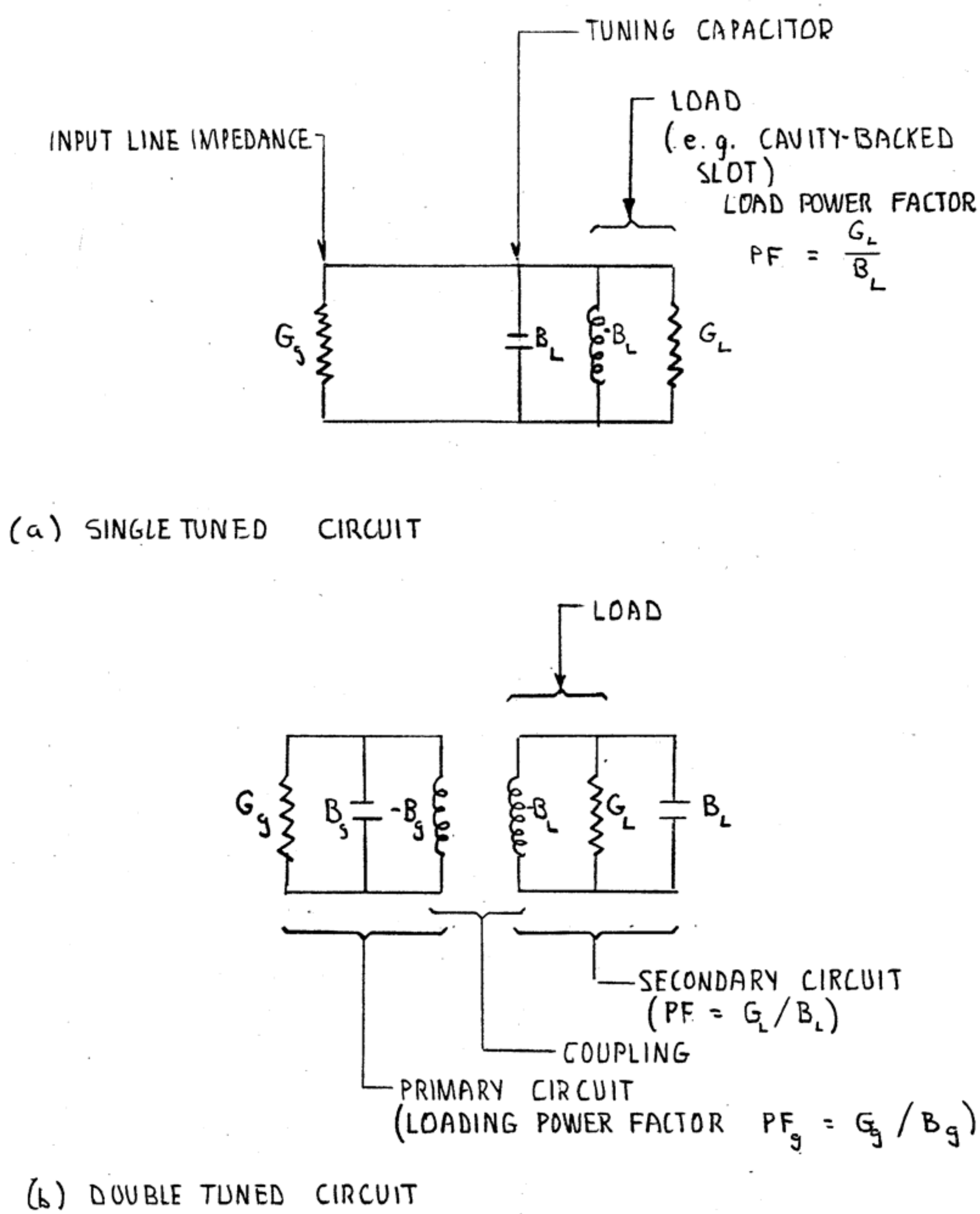

FIG. 1 SINGLE AND DOUBLE TUNED CIRCUITS.

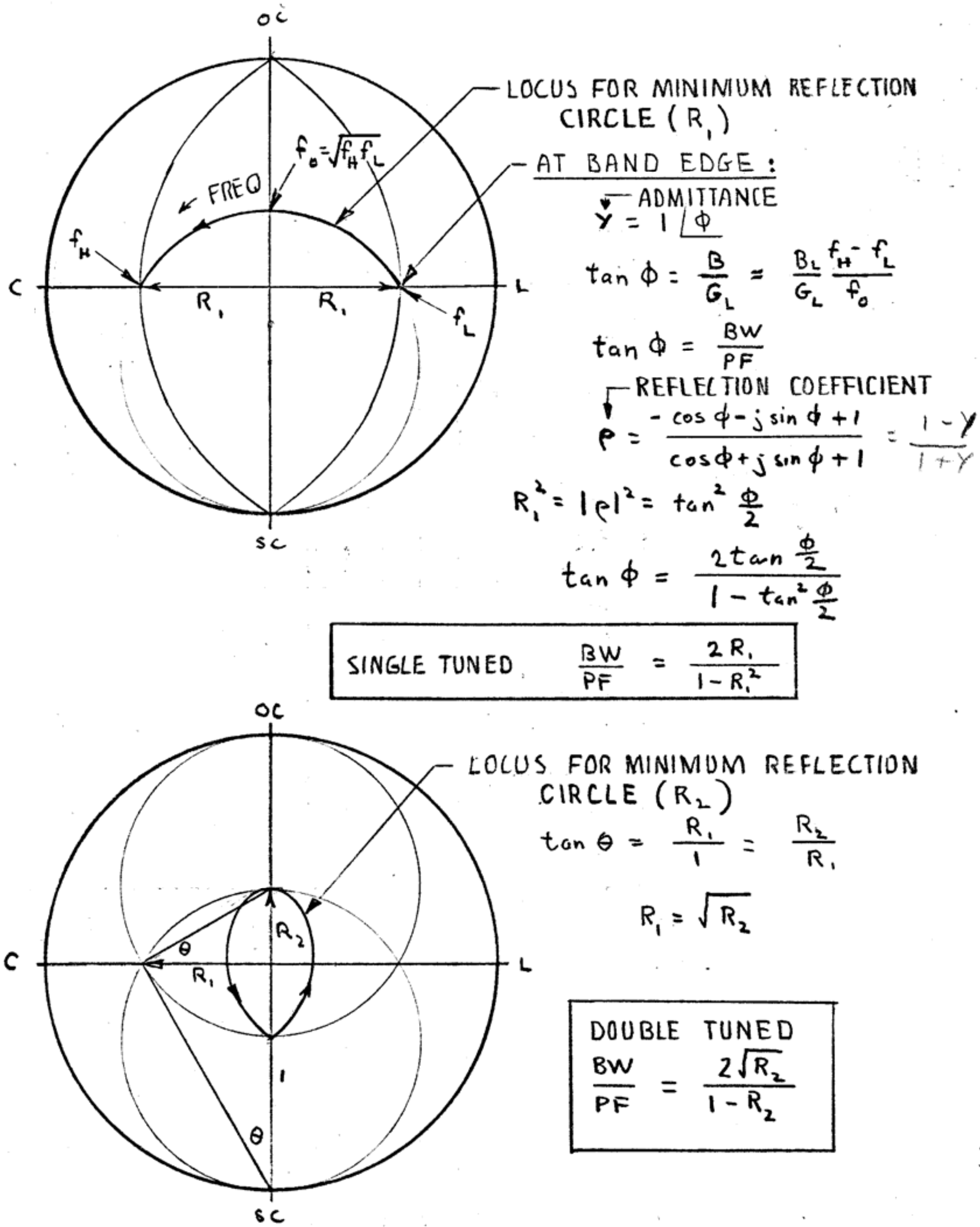

FIG. 2 — BANDWIDTH / POWER FACTOR RATIO AND MINIMUM REFLECTION CIRCLE FOR SINGLE AND DOUBLE TUNING.

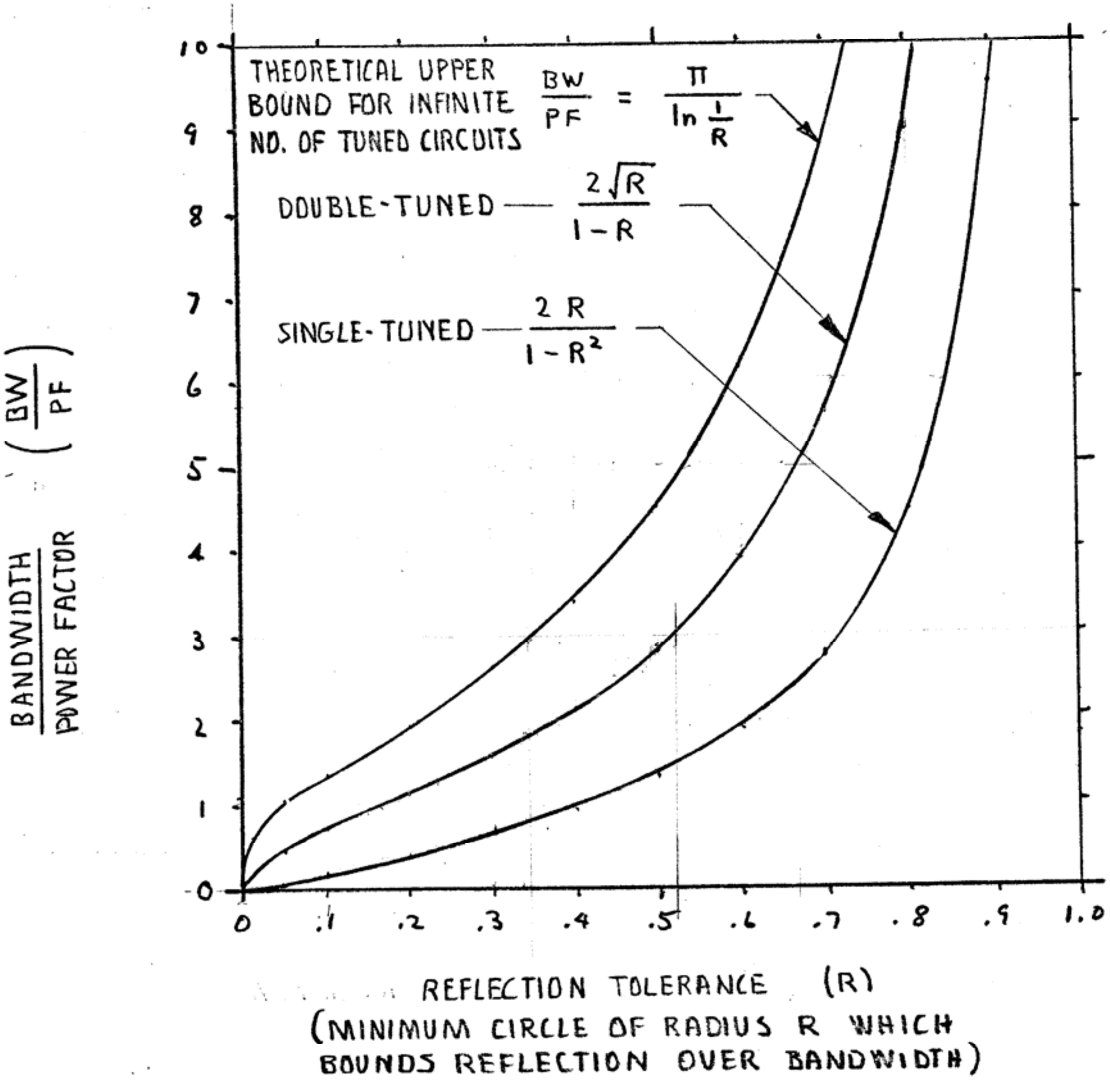

FIG. 3 - BANDWIDTH / POWER FACTOR VERSUS REFLECTION FOR MULTIPLE-TUNED CIRCUITS.

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TUNING — FOR R > 1/3

1 $$\frac{BW}{PF} = \frac{2R}{1-R^2} = \frac{2}{R^{-1}-R^{1}} = \frac{1}{\sinh\left(\ln\frac{1}{R}\right)} \approx \frac{1}{\ln\frac{1}{R}}$$

2 $$\frac{BW}{PF} = \frac{2\sqrt{R}}{1-R} = \frac{2}{R^{-\frac{1}{2}}-R^{\frac{1}{2}}} = \frac{1}{\sinh\left(\frac{1}{2}\ln\frac{1}{R}\right)} \approx \frac{2}{\ln\frac{1}{R}}$$

3 - - - - - - - - - - - - - - - - $$\frac{2\frac{1}{3}}{\ln\frac{1}{R}}$$

⋮

∞ $$\frac{BW}{PF} \leq \frac{\pi}{\ln\frac{1}{R}} \leq \frac{\pi}{\ln\frac{1}{R}}$$

FIG. 4 - BANDWIDTH/POWER FACTOR RATIO FOR R > 1/3

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DEPARTMENT OF ELECTRICAL ENGINEERING

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE, MASSACHUSETTS 02139

February 27, 1973

Mr. Alfred R. Lopez
Consulting Engineer
Hazeltine Corporation
Wheeler Laboratory
Greenlawn, New York 11740

Dear Mr. Lopez:

Please excuse my delay in replying to your letter of January 31, 1973. I find myself in the embarrassing position of remembering very little about my own work, since I have been away from the field of network theory, and particularly the area of my doctoral thesis since 1947. I have wandered from field to field ever since, but I never got back to where I started from. Thus, the questions in your letter would require, at this point, an inordinate amount of work. However, it seems to me, after looking over the pertinent part of my paper, that you should be able to check the values of your constants S_n, and the general equations given in your memorandum from equations 36, 37 and 38 in my paper, from which the curves in figure 19 were derived. I am sorry that I cannot face the task of making sense of my own work after a quarter of a century and carry out the analysis myself.

Since in your memorandum you refer to my RLE Technical Report, I am enclosing, herewith, a reprint of the same work which appeared in the Journal of the Franklin Institute; it may be a more convenient reference. Please convey my best regards and wishes to Dr. Wheeler.

Sincerely yours,

Robert M. Fano

Robert M. Fano
Ford Professor of Engineering and
Associate Department Head for
Computer Science and Engineering

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Enclosure

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