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Harold A. Wheeler's Antenna Design Legacy

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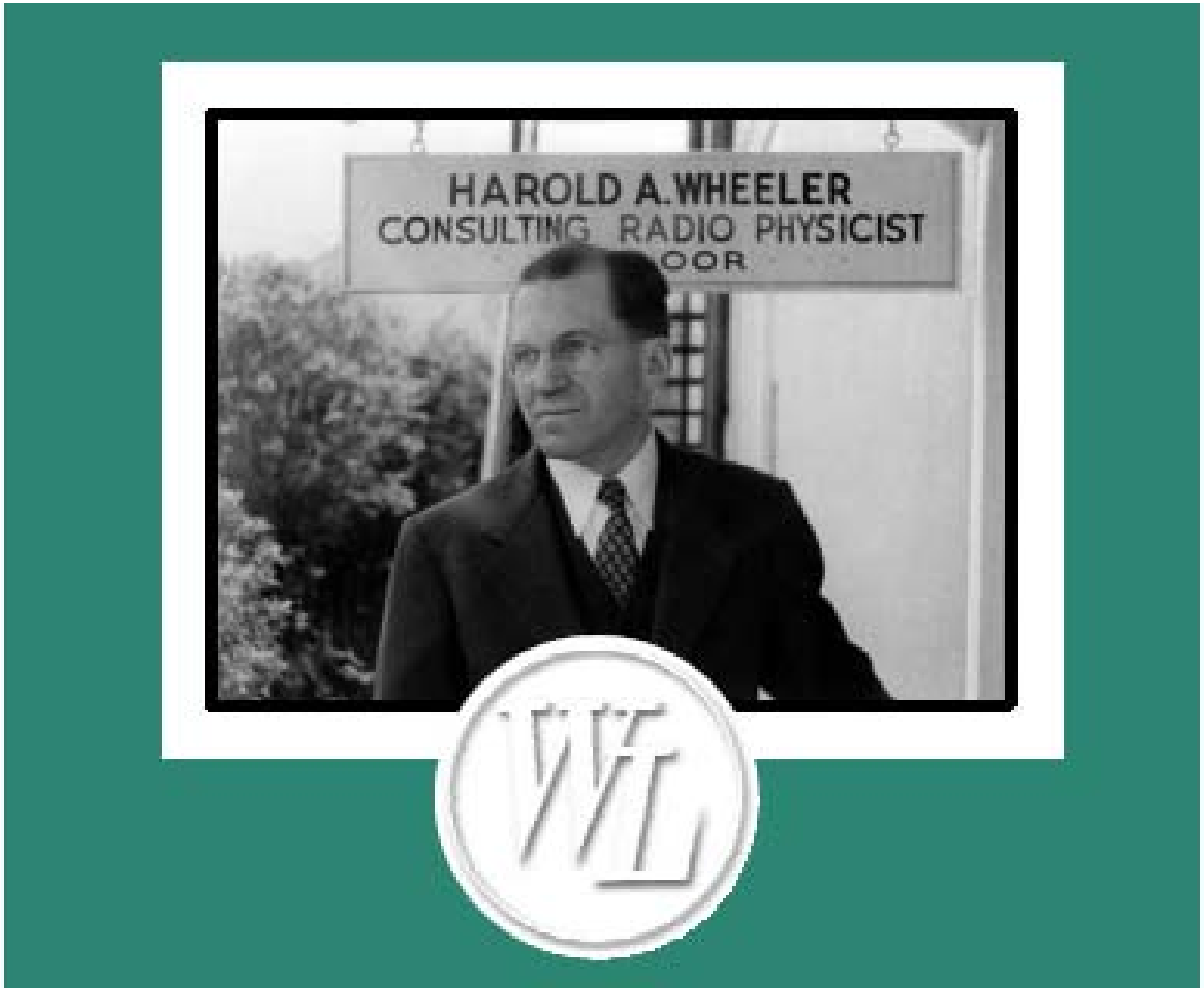
Outline of Presentation

- Harold A Wheeler (Some Highlights)
- Harold A. Wheeler's Antenna Design Legacy
  - Matching to a transmission line
  - Electrically small antenna
  - Planar array
- Electrically Small Antenna
  - "Small antenna behaves as a lumped element with some radiation"
- Matching To A Transmission Line
  - Art of impedance matching using the reflection chart as the primary tool
- Closing Remarks

Harold A Wheeler



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Harold A. Wheeler (Some Highlights)

- May 10, 1903 - April 25, 1996
- 1925 Invented automatic volume control (AVC)
- 1939 IRE Morris Liebmann Memorial Prize (paper on TV amplifiers)
- 1941 Development of mine detector used by WWII allied forces
- 1947 Started Wheeler Laboratories, inc.
- 1948 Chairman of IEEE (IRE) Long Island Section
- 1964 IEEE Medal of Honor (highest IEEE award)
- 1965-1968, Hazeltine, Chairman of the Board
- 180 US patents
- More than 90 publications over a span of 57 years
  - 1928 Automatic volume control for radio receiving sets
  - 1947 Fundamental limitations of small antennas
  - 1975 Small antennas
  - 1985 Antenna topics in my experience

Harold A. Wheeler's Antenna Design Legacy

- 1985 Paper, Antenna Topics in My Experience
  - Wideband Matching To A Transmission Line (1930s to 1980s)
    - The union of antenna impedance and circuit theory
    - Horizontal biconical dipole (6-18 MHz, 1936)
    - Vertical Monopole on a mast (157-187 MHz, 1943)
    - Double tuning on reflection chart
  - Small Antennas (1940s to 1980s)
    - 1947 "Behaves as a capacitor or an inductor with some radiation resistance" (lumped-element small antenna concept)
    - 1975 Effective volume concept
  - Planar Arrays (1960s)
    - Concept of infinite array (1948)
    - Relations from current sheet, scan angle (1964)
    - The impedance crater (1965)
    - The grating-lobe series (1965)
    - Array simulators in waveguide (1963)

Small Antenna Legacy

Small Antennas, 1947 Paper

$$Q < \frac{6\pi(\lambda/2\pi)^3}{Ab}$$

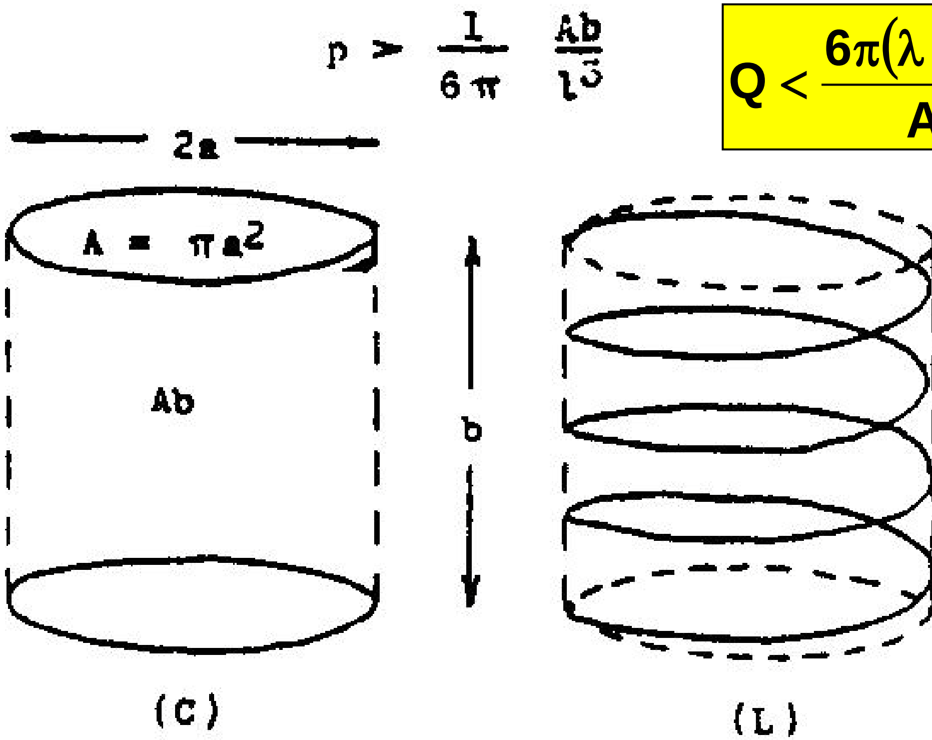


Fig. 1—Capacitor (*C*) and inductor (*L*) occupying equal cylindrical volumes.

Original figure appearing in Wheeler’s 1947 paper

Some Definitions (Wheeler)

Radianlength

The radianlength is the wavelength divided by 2π ($\lambda/2\pi$)

Radiansphere

The radiansphere is a sphere whose radius is the radianlength. It is the boundary between the near field and far field of a small antenna. Volume of radiansphere = $V_{RS} = 4/3\ \pi\ (\lambda/2\pi)^3$

Small Antenna

"The small antenna to be considered is one whose maximum dimension is less than the radianlength."

"An antenna within this limit of size can be made to behave as lumped capacitance or inductance, so this property is assumed."

Wheeler and Chu

Wheeler (1947 and 1975 Papers)

$$Q = \frac{9}{2}\frac{V_{RS}}{V_E} = \frac{9}{2}\frac{V_{RS}}{kV_{OC}}$$

V_E = Effective Volume

k = Effective Volume Factor

V_{OC} = Wheeler Occupied Volume

For Cylindrical Antenna: V_{OC} = Ab
Where A = area of cylinder base and
b = height of cylinder

Wheeler's formulas yield accurate values, and are invaluable for the design of small antennas.

Chu (1948 Paper)

Q_{Chu} = LOWER BOUND ON Q

$$Q_{Chu} = \frac{V_{RS}}{V_{Chu}} \quad \text{for} \quad V_{Chu} << V_{RS}$$

V_{Chu} = Chu Volume
= Volume of sphere whose diameter is the maximum dimension of the small antenna

Chu's lower bound assumes no stored energy within the sphere and can only be realized in theory. It is a theoretical reference point.

Wheeler's Formula for Capacitor Antenna

$$Q_{\text{Wheeler(Capacitor)}} = 6\pi \frac{\left(\frac{\lambda}{2\pi}\right)^3}{\pi a^2 b} \frac{1}{k_a} = \frac{9}{2} \frac{V_{RS}}{V_E} = \frac{9}{2} \frac{V_{RS}}{V_{OC}} \frac{k_{SC} + \varepsilon_r - 1}{{k_{SC}}^2}$$

Where $a = \text{Disc radius}$

$b = \text{Distance between discs, dipole length}$

$k_a = k_{SC} k_{FC} = \text{Effective volume factor}$

$k_{SC} = \text{Shape factor} > 1$

$$k_{SC} = 1 + \frac{4}{\pi} \frac{b}{a}$$

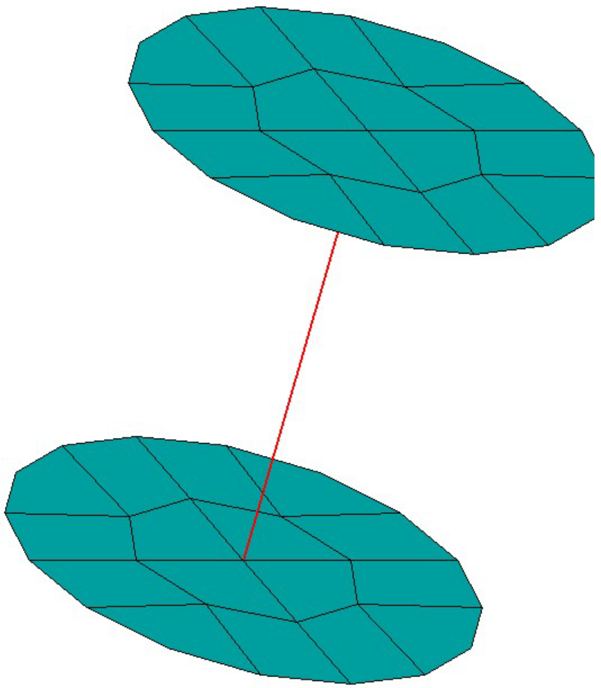
$k_{FC} = \text{Fill factor}$

$$k_{FC} \approx \frac{k_{SC}}{k_{SC} + \varepsilon_r - 1} \qquad \text{For } \frac{b}{a} < 2$$

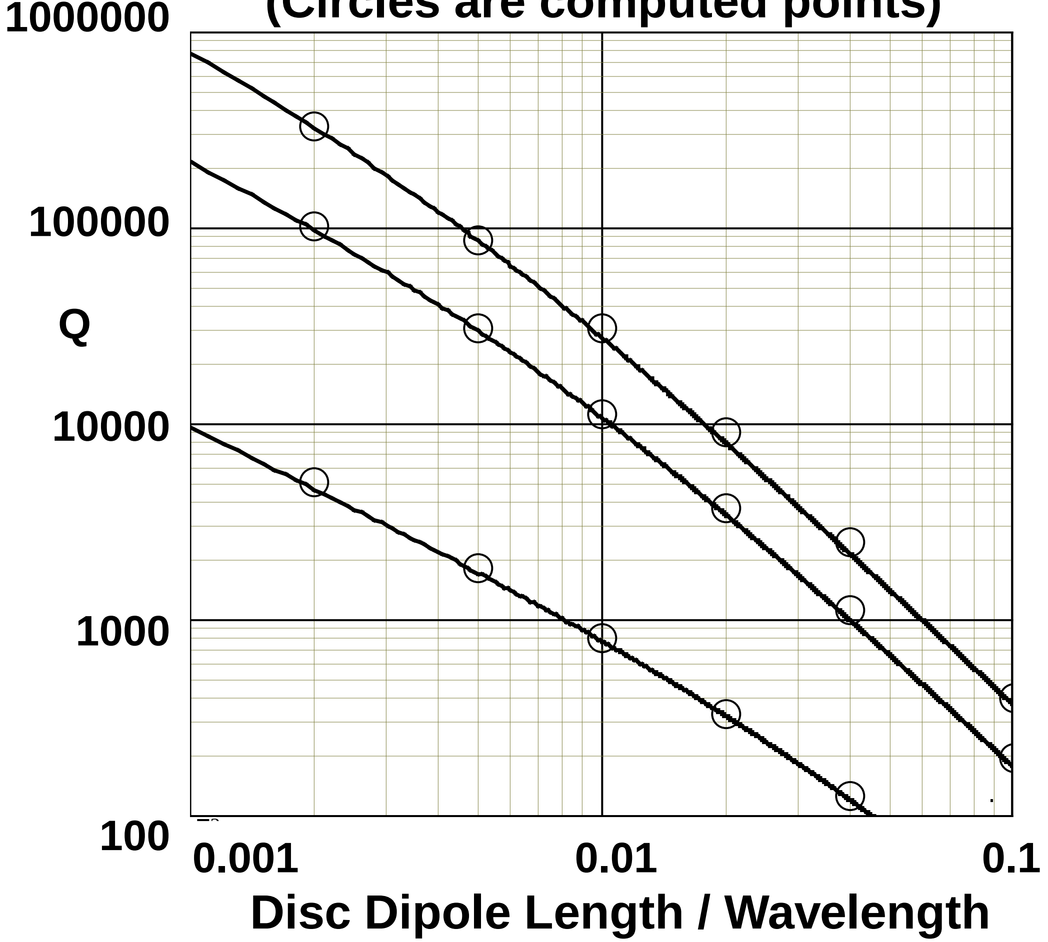
$\varepsilon_r = \text{Relative permitivity of fill (core) material}$

Validation of Wheeler's Formula for Capacitor

Disc Dipole Computer Model



Q Versus Disc Dipole Length
Disc Radius / λ = 0.005 top, 0.01, and 0.05
(Circles are computed points)



Wheeler's Formula for Inductor Antenna

$$Q_{\text{Wheeler(Inductor)}} = 6\pi \frac{\left(\frac{\lambda}{2\pi}\right)^3}{\pi a^2 b}\frac{1}{k_b} = \frac{9}{2}\frac{V_{RS}}{V_E} = \frac{9}{2}\frac{V_{RS}}{V_{OC}}\frac{k_{SL} + 1/\mu_r - 1}{k_{SL}^{\,2}}$$

Where $a = \text{Loop radius}$

$b = \text{Loop axial length}$

$k_b = k_{SL}k_{FL} = \text{Effective volume factor}$

$k_{SL} = \text{Shape factor} > 1$

$k_{SL} \approx 1 + 0.9\frac{a}{b}$ Somewhat less than this value if $b < a$

$k_{FL} = \text{Fill factor}$

$k_{FL} \approx \frac{k_{SL}}{k_{SL} + 1/\mu_r - 1}$ For $\frac{b}{a} > 2$

$\mu_r = \text{Relative permeability of fill (core) material}$

Validation of Wheeler's Formula for Inductor

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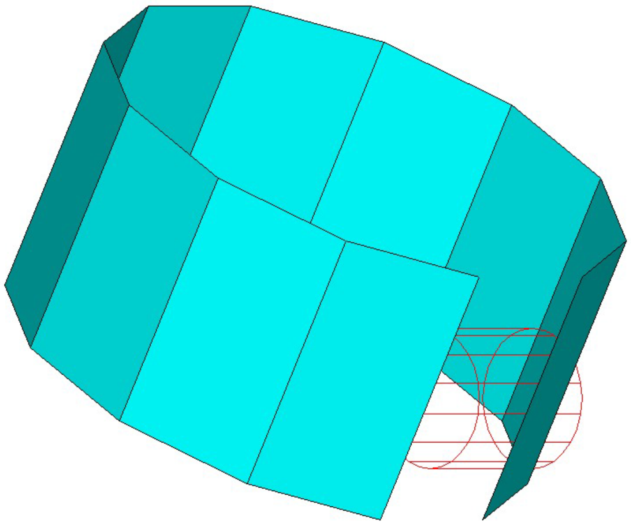
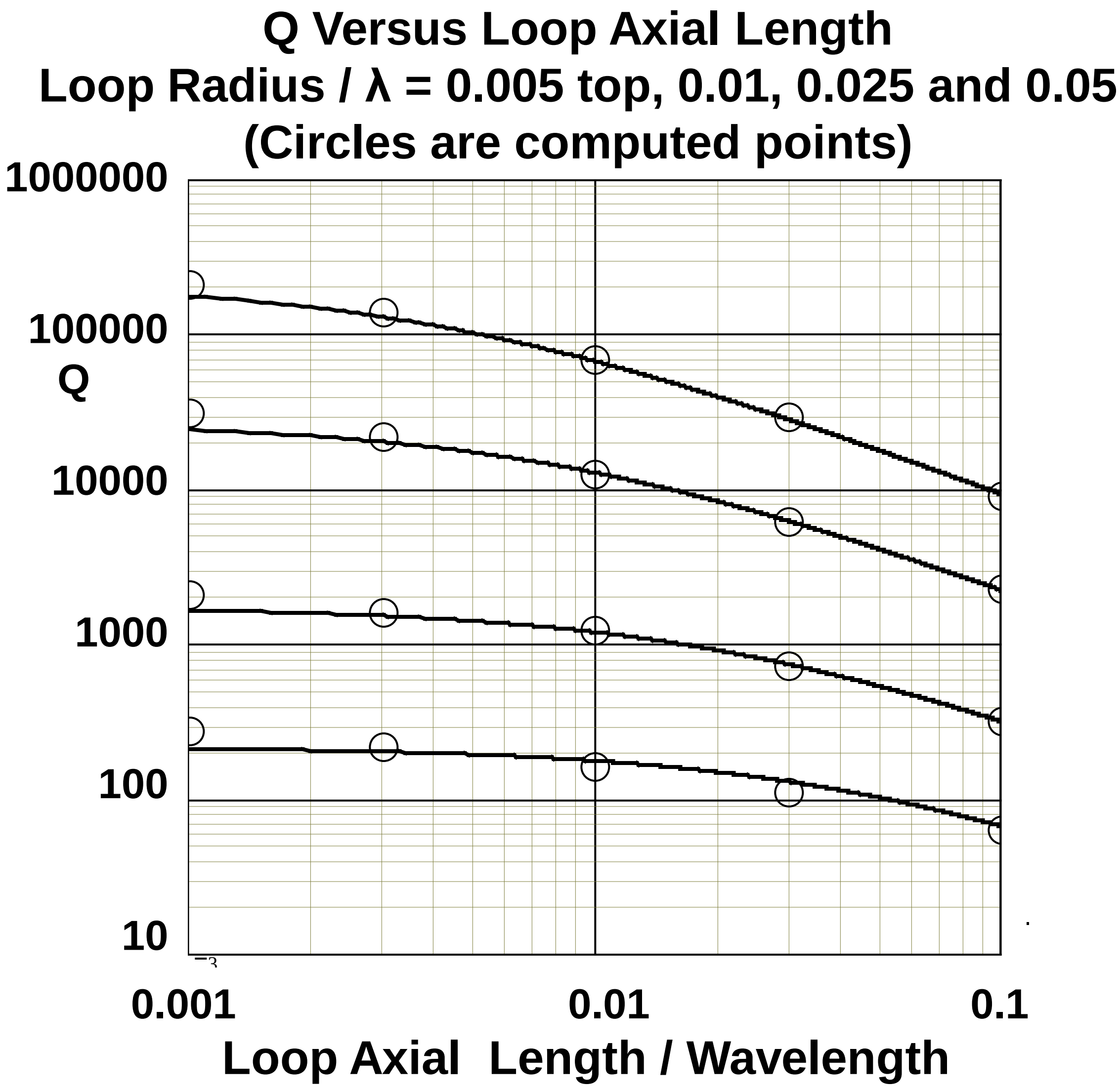
Loop Inductor Computer Model





Q Dependence on ε_r and μ_r

Capacitor Antenna:

$$Q_{Wheeler} = \frac{9}{2}\frac{V_{RS}}{V_{OC}}\frac{k_{SC} + \varepsilon_r - 1}{k_{SC}^2}$$

ε_r = Relative Permitivity

$k_{SC} > 1$

Q increases with increasing ε_r

Inductor Antenna:

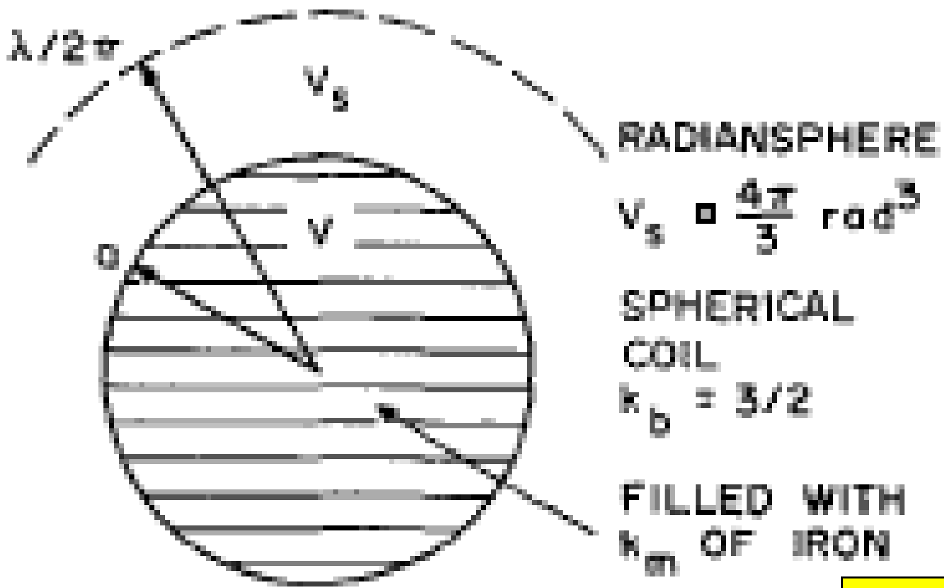
$$Q_{Wheeler} = \frac{9}{2}\frac{V_{RS}}{V_{OC}}\frac{k_{SL} + 1/\mu_r - 1}{k_{SL}^2}$$

μ_r = Relative Permeability

$k_{SL} > 1$

Q decreases with increasing μ_r

Wheeler's Implementation of Chu's Lower Bound on Q



$$\frac{\text{EFFECTIVE VOLUME}}{\text{COIL VOLUME}} = \frac{1}{1 + 2/k_m}$$

$$\frac{9}{2}\,\frac{1}{1+2/k_m}$$

Two Typos

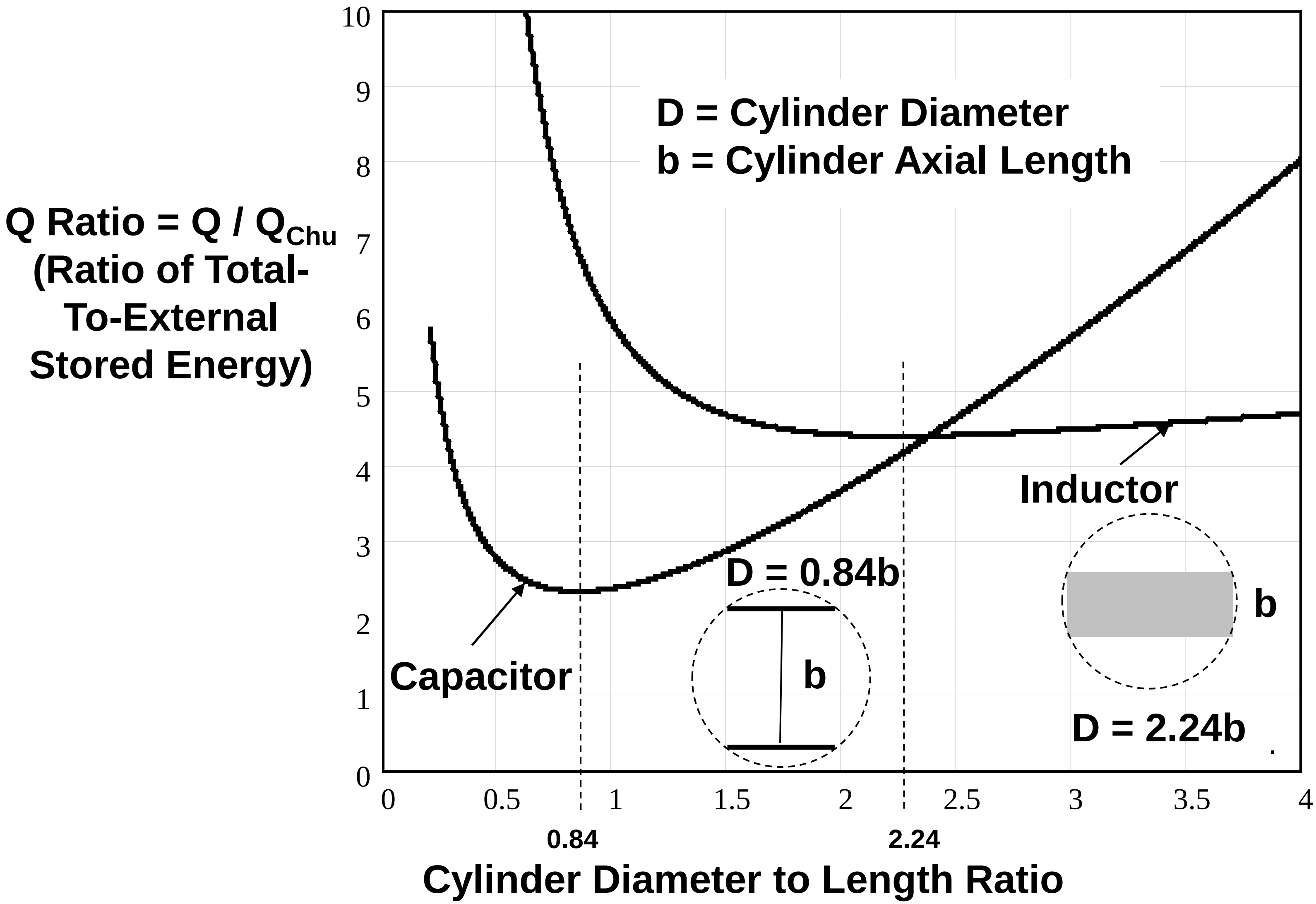
$$p_m = \left(\frac{2\pi a}{\lambda}\right)^3 \frac{1}{1 + 2/k_m} = \frac{V}{V_r}\,\frac{1}{1 + 2/k_m} < \frac{V}{V_r}$$

$$V_r = V_S$$

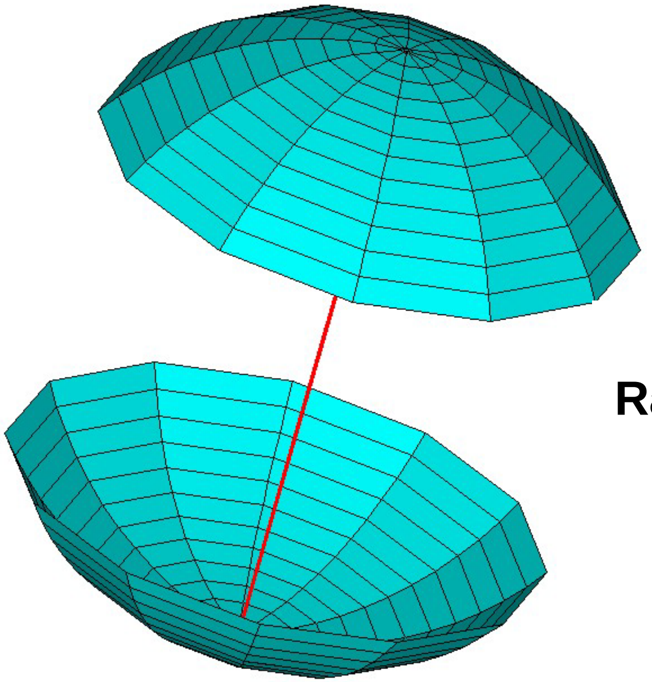
Fig. 5. Spherical coil with magnetic core.

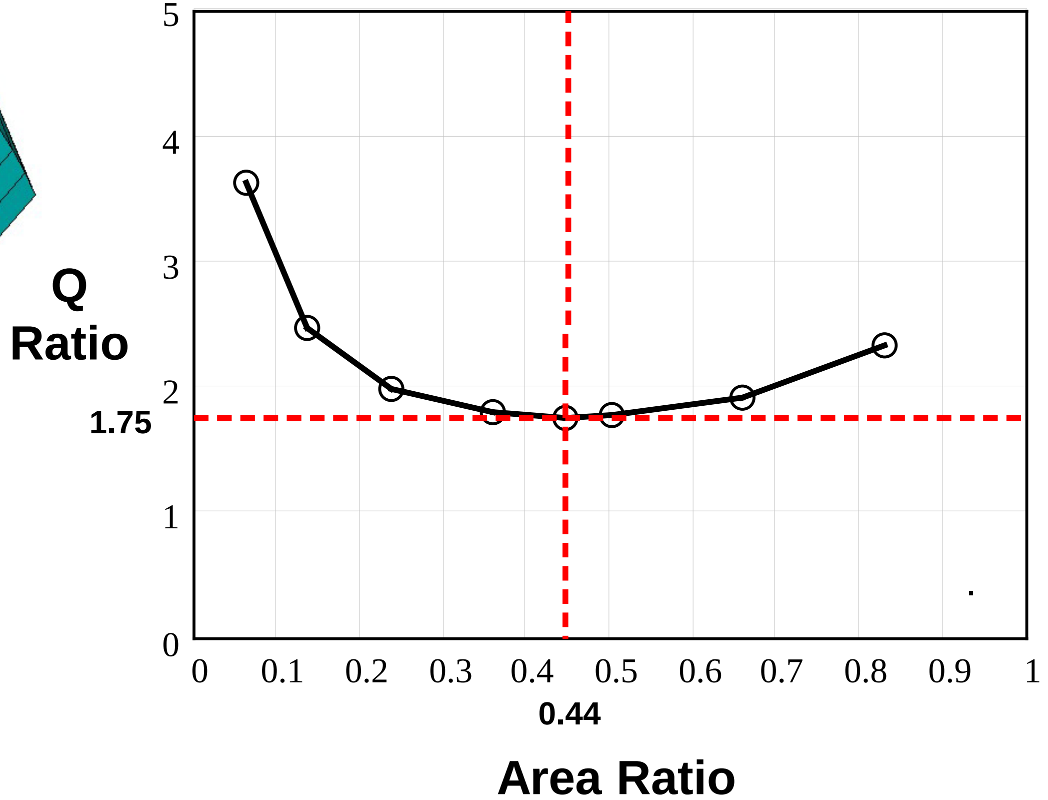
Original figure first appearing in Wheeler's 1975 paper

Optimum Shape for Cylindrical Antennas



Optimum Spherical-Cap Dipole





Wheeler Prediction 1985: Minimum Q when cap area about ½ sphere area

Lower Bound for Q of Capacitor (Electric) Small Antennas

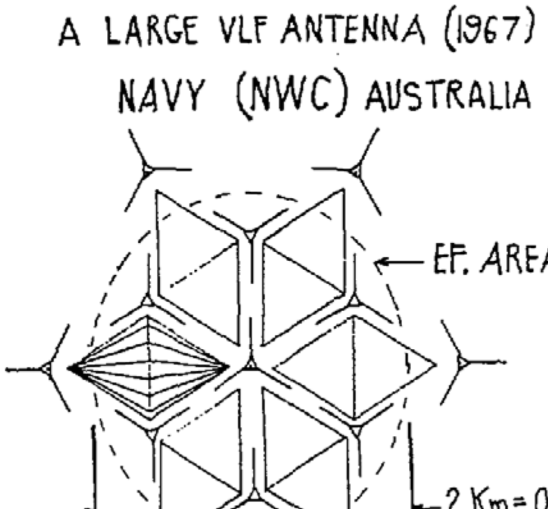
Small Antenna Q Ratios

Q Ratio = Antenna Q / Chu Lower-Bound Q

Antenna Type	Q Ratio = Q/Q_{Chu}
Spherical Inductor, $\mu_r = \infty$	1.0
Spherical Inductor, $\mu_r = 1$	3.0
Cylindrical Inductor $\mu_r = 1$, Diameter / Length = 2.24	4.4
Disc Dipole Diameter / Length = 0.84	2.4
Spherical-Cap Dipole	1.75

It is believed that the Q for the spherical-cap dipole is the lower bound for the Q of the capacitor (electric) antenna

"The Largest Antenna in the World is a Small Antenna"



Wheeler, 1985

Antenna located at North West Cape, Australia

Operates at 15.5 KHz

Diameter of outer tower circle is 1.7 miles (2.7Km)

Tower height is 1300 Ft. (390m)

Radiation Q = 435

Radiation Bandwidth = 36 Hz

Resonance Bandwidth = 134 Hz

Antenna Impedance Matching Legacy

Impedance Matching

- 1935 Antenna perceived as a circuit element. Circuit theory applied to impedance matching antenna to transmission line.
- Reflection Chart (Tool for Impedance Matching)
  - 1936 Paper "Doublet antennas and transmission lines" (Reflection chart on resistance and reactance coordinates)
  - 1940 Reflection Charts (Smith, Carter and Wheeler)
  - Wheeler developed the art of impedance matching using the reflection chart as the primary tool (Smith and Carter charts were used as under-lays for the Wheeler reflection chart in graphical solutions to impedance matching problems)
- Optimum impedance matching of single-tuned and double-tuned antennas

Antenna Bandwidth Definitions

Radiation Bandwidth (Ratio) = 1/Q = $(f_H - f_L)/f_0$ $f_0 = \sqrt{f_H f_L}$

= Antenna 3 dB Bandwidth

= Antenna Resonance Bandwidth

For an antenna tuned to resonance, f_H and f_L are the high and low frequencies where the magnitude of the reactance is equal to the resistance (f_0 is the resonant frequency)

(Impedance) **Matching Bandwidth** (Ratio) = B = $(f_H - f_L)/f_0$

f_H and f_L are the high and low frequencies of a frequency band over which a specified magnitude of reflection coefficient (or VSWR) is not exceeded

$B(\Gamma) = f(\Gamma)/Q$ where Γ is the maximum reflection magnitude

Matching Factor = $QB(\Gamma) = f(\Gamma)$

Matching Bandwidth is equal to the **Radiation Bandwidth** multiplied by the **Matching Factor**

Optimum Impedance Matching: Wheeler and Fano

Wheeler

$$QB = \frac{2\Gamma^{\frac{1}{n}}}{1-\Gamma^{\frac{2}{n}}}$$

$$n = 1 \text{ and } 2$$

Single Tuned n = 1
Double Tuned n = 2
Infinite Tuned n = ∞

Fano

$$QB = \frac{2\sin\left(\frac{\pi}{2n}\right)}{\sinh(a) - \sinh(b)}$$

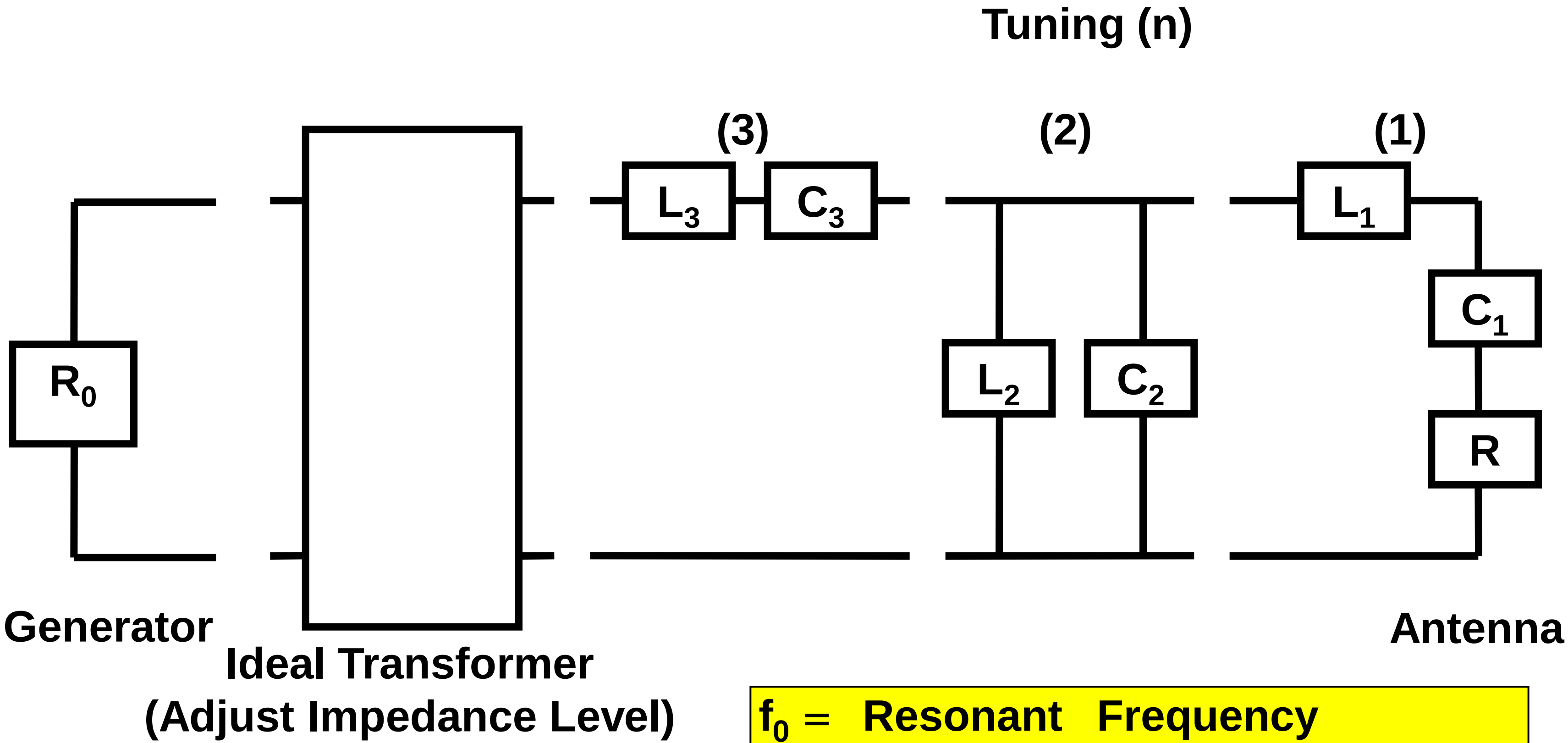
$$\frac{\tanh(na)}{\cosh(a)} = \frac{\tanh(nb)}{\cosh(b)}$$

$$\frac{\cosh(nb)}{\cosh(na)} = \Gamma$$

$$n = 1, 2, 3 \ldots\ldots\ldots \infty$$

It is remarkable that the Wheeler and Fano equations are in exact agreement (n = 1, n = 2), considering their radical difference in form.

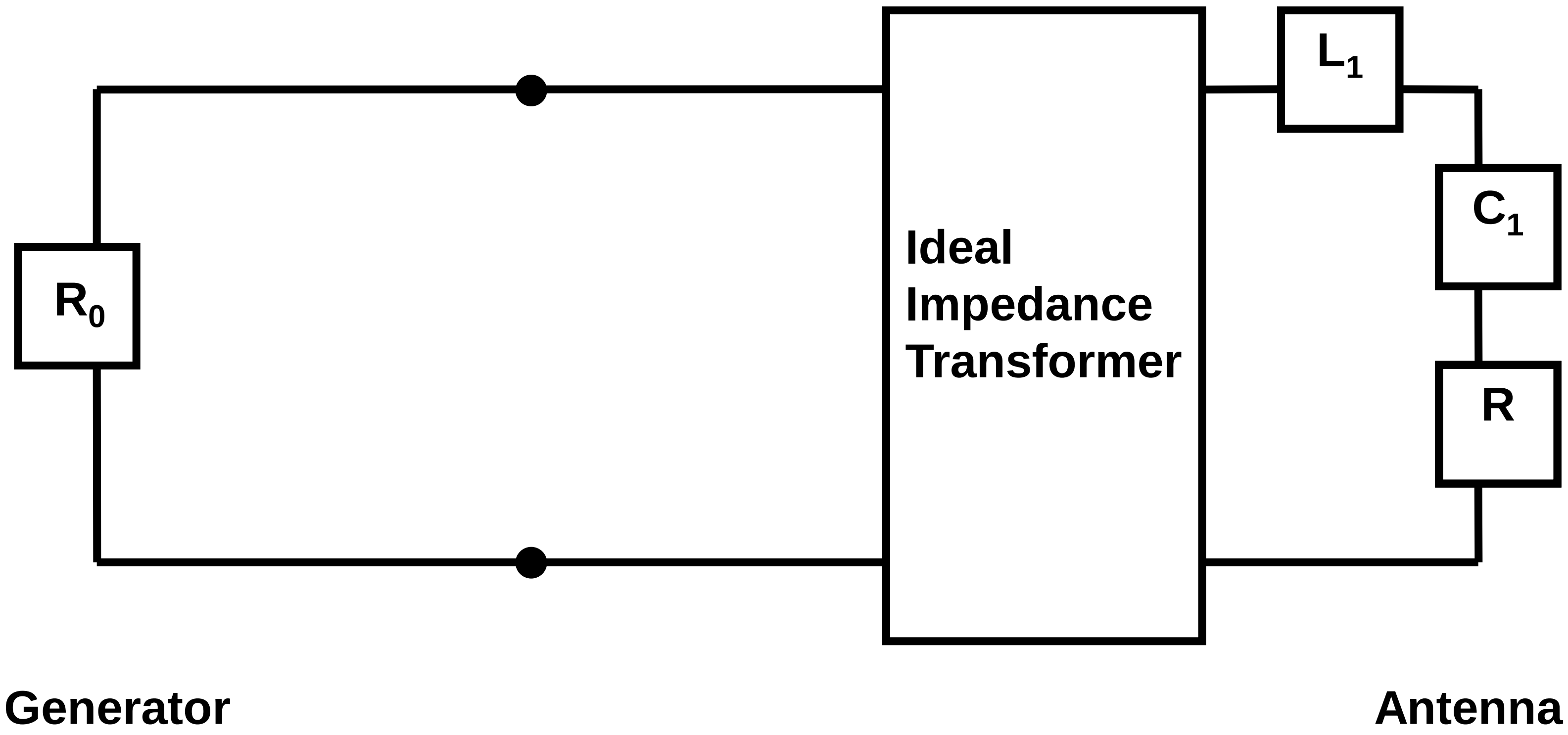
Multiple Tuning Impedance Matching Network



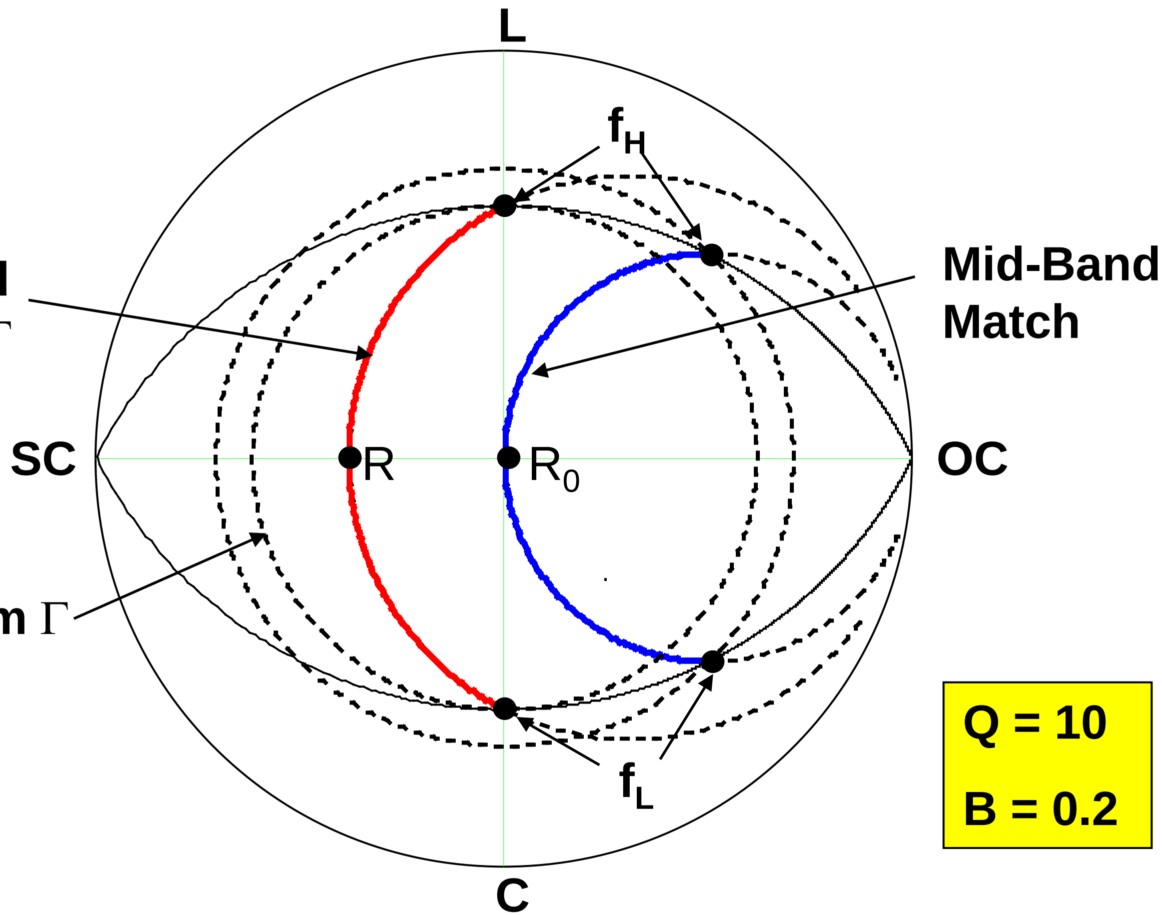
f_0 = Resonant Frequency

$$f_0 = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} = \frac{1}{2\pi\sqrt{L_3C_3}}$$

Wheeler Single Tuning



Single Tuning Reflection Chart



Single Tuning Equation

$$Z(f_H) = R + j\omega_0 L_1\left(\frac{f_H}{f_0} - \frac{f_0}{f_H}\right) = R\left(1 + \frac{j\omega_0 L_1}{R}\left(\frac{f_H}{f_0} - \frac{f_L}{f_0}\right)\right)$$

$$z(f_H) = \frac{Z(f_H)}{R_0} = \frac{R}{R_0}(1 + jQB) = \exp(j\phi) = \cos(\phi) + j\sin(\phi)$$

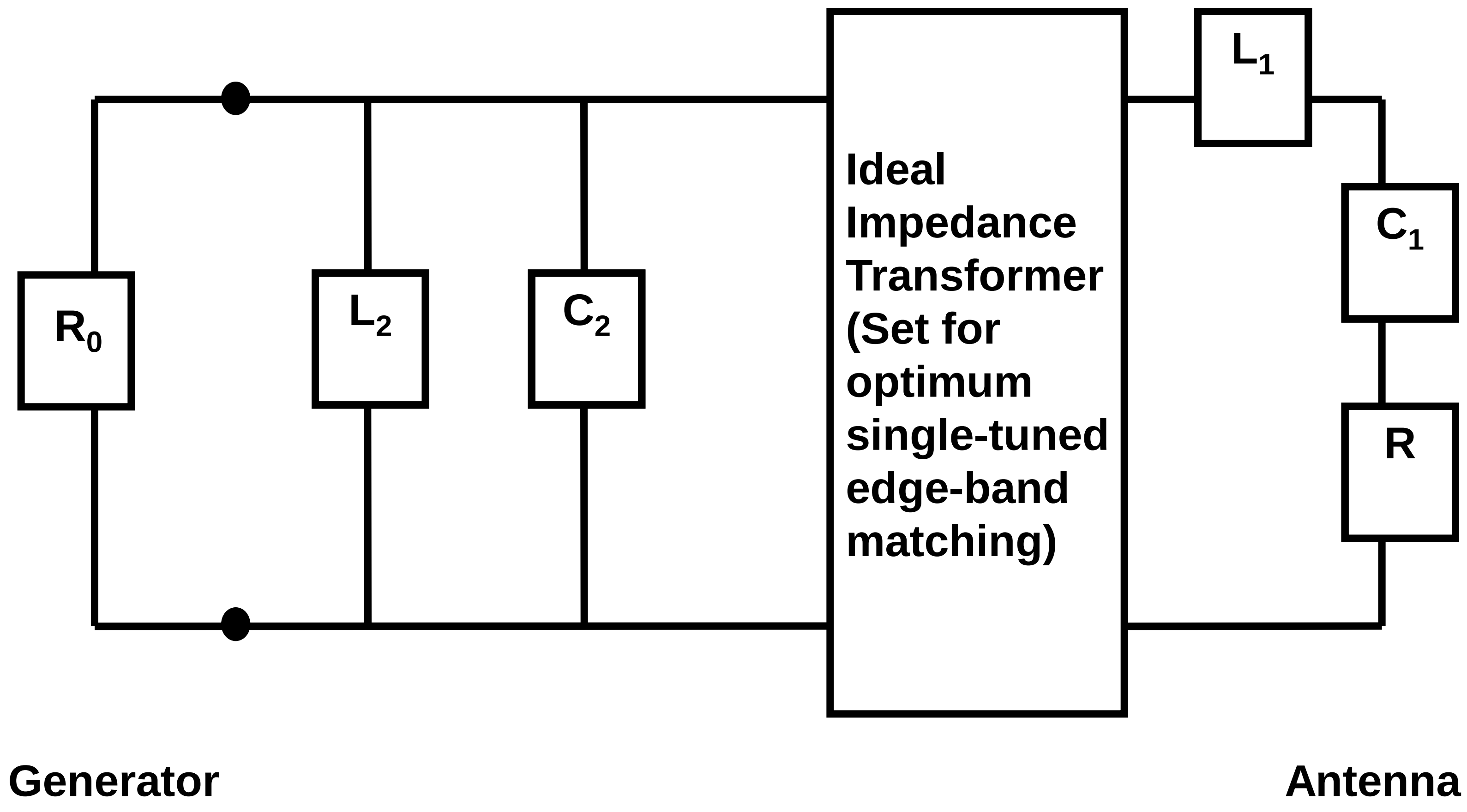
$$\left|\frac{R}{R_0}(1 + jQB)\right| = 1 \qquad \frac{R_0}{R} = \sqrt{1 + (QB)^2} \qquad \tan(\phi) = QB$$

$$\Gamma = \left|\frac{z(f_H) - 1}{z(f_H) + 1}\right| = \sqrt{\frac{(\cos(\phi) - 1)^2 + \sin(\phi)^2}{(\cos(\phi) + 1)^2 + \sin(\phi)^2}} = \tan(\phi/2)$$

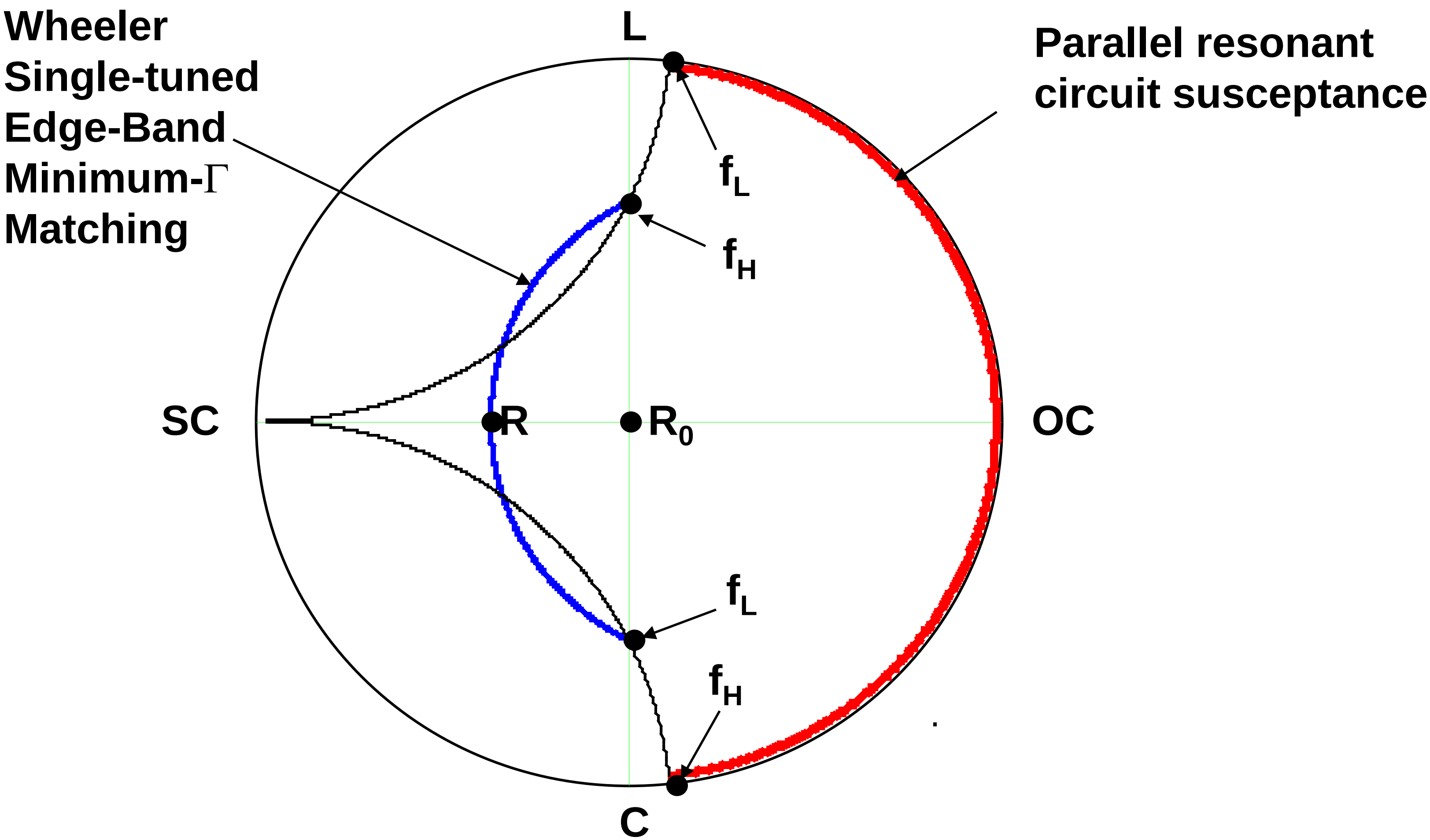
$$\tan(\phi) = \frac{2\tan(\phi/2)}{1 - \tan(\phi/2)^2}$$

$$QB = \frac{2\Gamma}{1 - \Gamma^2} = \frac{2\Gamma^{\frac{1}{n}}}{1 - \Gamma^{\frac{2}{n}}} \qquad n = 1$$

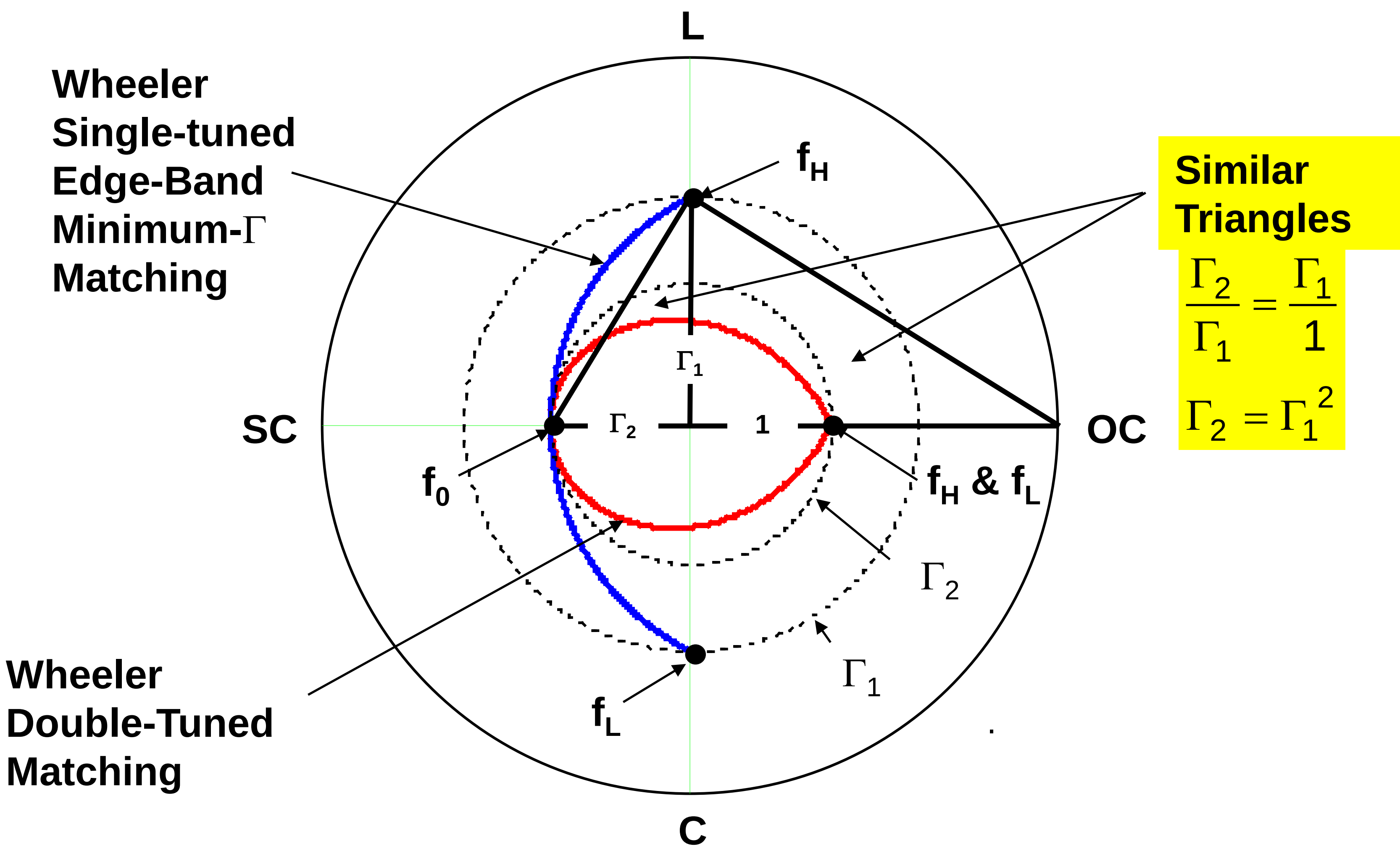
Wheeler Double Tuning



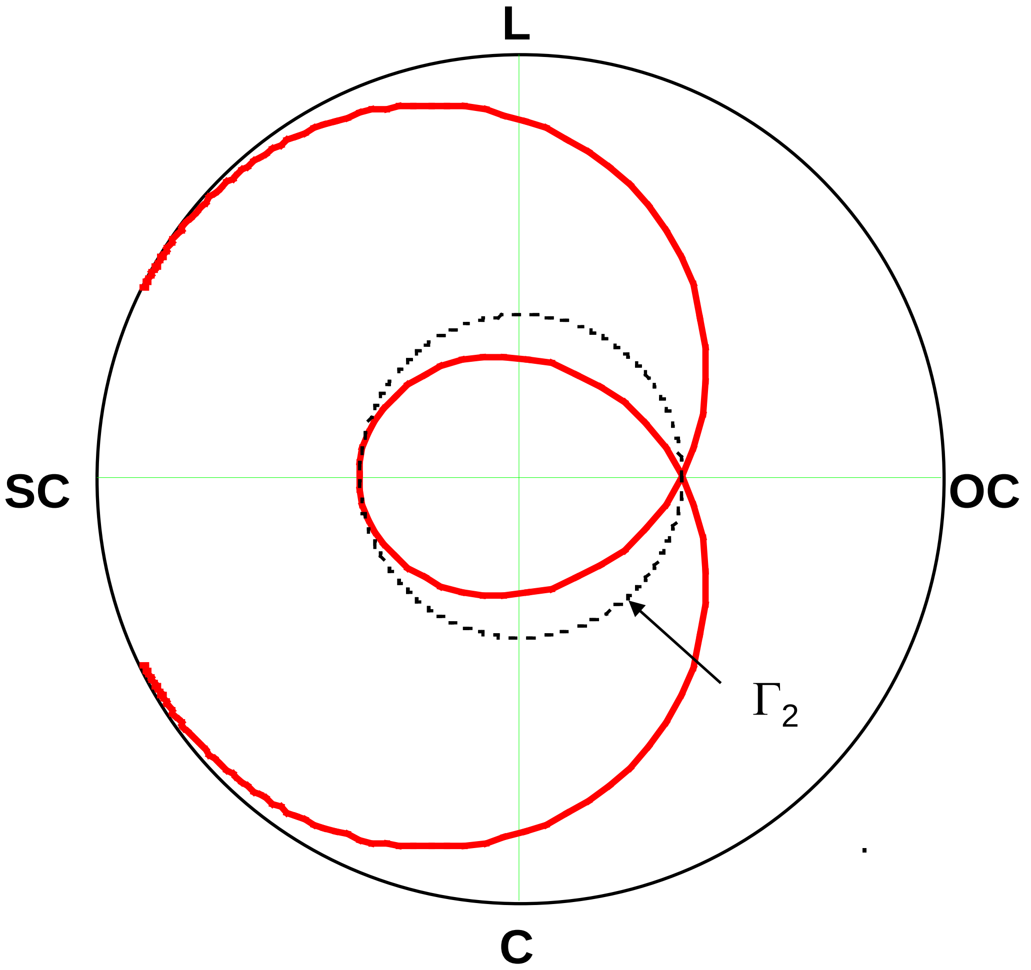
Double Tuning Reflection Chart



Double Tuning Reflection Chart (continued)



Double Tuning Reflection Chart (continued)



Q of Parallel Resonant Circuit

$$b_2(f_L) = \frac{\omega_0 C_2}{G_0}\left(\frac{f_L}{f_0} - \frac{f_0}{f_L}\right) = -Q_2 B$$

Normalized Susceptance of Parallel Resonant Circuit at f = f_L

$$= \mathrm{Im}\left(\frac{1}{z_1(f_H)}\right) = \mathrm{Im}\left(\frac{R_0}{R(1+jQB)}\right)$$

Normalized Susceptance of Series Resonant Circuit at f = f_H

$$= \mathrm{Im}\left(\frac{\sqrt{1+(QB)^2}}{1+jQB}\frac{1-jQB}{1-jQB}\right) = \mathrm{Im}\left(\frac{1-jQB}{\sqrt{1+(QB)^2}}\right)$$

$$Q_2 = \frac{Q}{\sqrt{1+(QB)^2}}$$

Double Tuning Equation

$$\text{Start with the single tuned case:}$$

$$QB = \frac{2\Gamma_1}{1 - \Gamma_1^2}$$

$$\Gamma_1 = \text{Maximum single-tuned reflection magnitude}$$

$$\Gamma_2 = \Gamma_1^2$$

$$\Gamma_2 = \text{Maximum double-tuned reflection magnitude}$$

$$\text{Double Tuned:}$$

$$QB = \frac{2\sqrt{\Gamma_2}}{1 - \Gamma_2} = \frac{2\Gamma^{\frac{1}{n}}}{1 - \Gamma^{\frac{2}{n}}} \quad n = 2$$

Impedance Matching Formulas - Summary

Impedance Matching Circuit	Impedance Matching Factor (QB)	QB for V = VSWR = 2
Single-tuned mid-band match (Non Fano)	$QB = \frac{2\Gamma}{\sqrt{1-\Gamma^2}} = \frac{V-1}{\sqrt{V}}$	QB = 0.707
Single-tuned edge-band matching (Wheeler-Fano, n = 1)	$QB = \frac{1}{\sinh\left(\ln\left(\frac{1}{\Gamma}\right)\right)} = \frac{2\Gamma}{1-\Gamma^2} = \frac{V^2-1}{2V}$	QB = 0.750
Double-tuned matching (Wheeler-Fano, n = 2)	$QB = \frac{1}{\sinh\left(\frac{1}{2}\ln\left(\frac{1}{\Gamma}\right)\right)} = \frac{2\sqrt{\Gamma}}{1-\Gamma} = \sqrt{V^2-1}$	QB = 1.732
Triple-tuned matching (Lopez-Fano, n = 3)	$QB = \frac{1}{b_3 \sinh\left(\frac{1}{a_3}\ln\left(\frac{1}{\Gamma}\right)\right) + \frac{1-b_3}{a_3}\ln\left(\frac{1}{\Gamma}\right)}$ $a_3 = 2.413$ $b_3 = 0.678$	QB = 2.146
Infinite-tuned matching (Fano-Bode, n = ∞)	$QB = \frac{\pi}{\ln\left(\frac{1}{\Gamma}\right)} = \frac{\pi}{\ln\left(\frac{V+1}{V-1}\right)}$	QB = 2.860

Rule of thumb for maximum achievable matching bandwidth (B_{MaxA})
If V > 2 then $B_{MaxA} \approx V / Q$

Closing Remarks

- **Wheeler's Small Antenna Legacy**
  - **Radianlength**
  - **Radiansphere**
  - **Small Antenna Lumped-Element Concept**
  - **Accurate Formulas for Q of Small Antennas**
  - **Relationship to Chu's Lower Bound on Q of Small Antennas**
- **Wheeler's Antenna Impedance Matching Legacy**
  - **Antenna perceived as a circuit element. Circuit theory applied to impedance matching antenna to transmission line.**
  - **Reflection Chart (Tool for Impedance Matching)**
  - **Simple formulas relating the Q-Bandwidth product to the maximum reflection magnitude**
  - **Wheeler and Fano contributions provide a complete picture of impedance matching limitations**

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References

[1] H. A. Wheeler, “Antenna Topics in My Experience,” IEEE Trans. Antennas Propagat.,Vol. AP-33, No. 2, Feb. 1985, pp. 144-151.

[2] A. R. Lopez, “Fundamental Limitations of Small Antennas: Validation of Wheeler’s Formulas,” IEEE Antennas Propagat. Mag., Vol. 48, No. 4, Aug. 2006, pp. 28-36.

[3] A. R. Lopez, “Review of Narrowband Impedance-Matching Limitations,” Antennas Propagat. Mag., Vol. 46, No. 4, Aug. 2004, pp. 88-90.

[4] A. R. Lopez, “Wheeler and Fano Impedance Matching,” IEEE Antennas Propagat. Mag., to be published.

[5] H. A. Wheeler, “Fundamental Limitations of Small Antennas,” Proc. IRE, 35, Dec. 1947, pp. 1479-1484.

[6] L. J. Chu, “Physical Limitations of Omni-Directional Antennas,” J. Appl. Phys., 19, Dec. 1948, pp. 1163-1175.

[7] H. A. Wheeler, “Reflection Charts Relating to Impedance Matching,” IEEE Trans. Microwave Theory Tech., Vol. MTT-32, No. 9, Sep. 1984, pp. 1008-1021.

[8] R. M. Fano, “Theoretical Limitations of the Broadband Matching of Arbitrary Impedances,” J. Franklin Institute, 249, Jan, 1950, pp. 57-83, Feb. 1950, pp. 139-154.

[9] A. R. Lopez, Relevant papers available at http://www.arlassociates.net