Communications

Line-Source Excitation for Maximum Aperture Efficiency with Given Sidelobe Level

Abstract—An unusual family of line-source aperture excitations are described which provide maximum aperture efficiency for a given sidelobe level. The emphasis is on the direct description of the excitation functions. The simplicity of the new excitation provides insight into the role of certain excitation components with respect to aperture utilization and sidelobe level. The proposed excitations are evaluated and are found to be comparable with the most efficient excitations previously described.

I. Introduction

A basic and intriguing problem which has been of much interest to the antenna design engineer is the question of what one line-source excitation yields maximum gain or maximum aperture efficiency for a specified restriction on the sidelobe level.[1] The author is not aware of any rigorous solution to this problem. This communication presents a new viewpoint; it describes another family of excitations which, for practical purposes, is a simple solution to the problem. It differs from previous approaches which concentrate on the pattern function; here the emphasis is on the description of the excitation function for maximum aperture efficiency. It also differs by virtue of its simplicity of formulation and computation.

II. Problem Background

For the purposes of this discussion, it is assumed that the line source is large in terms of wavelengths and that the excitation is equiphase (without any reversals, so supergain effects are excluded). Aperture efficiency is defined in accordance with the [11]:

$$\text{AE} = \frac{\text{directivity for any excitation}}{\text{directivity for uniform excitation}} = \frac{|\int f(x)\,dx|^2}{L\int f^2(x)\,dx}$$

$f(x)$ = line-source excitation (voltage or current)
L = length of line source.

The objective is to maximize AE with the sidelobe level specified.

The historical approach to the problem starts with the work of Dolph [1]. He determined the line-source array excitation coefficients for minimum beamwidth with all sidelobes at a given level. He related these coefficients to the Chebyshev polynomials.

van der Maas [2], [7], in attempting to simplify the calculation of a Dolph–Chebyshev array of many elements, discovered, by a limiting process, the excitation function for a continuous line aperture which has minimum beamwidth for a given sidelobe level. Fig. 1(a) shows this excitation and the corresponding pattern function: $I_1(y)$ is a modified Bessel function of the first kind and of order one. It is noted that, because an impulse is required at each edge, the excitation is unrealizable. (Campbell [6] published this Fourier-transform pair in 1928.)

Taylor [3], restricting himself to excitations which are analytic everywhere between the edges of a continuous aperture, modified the pattern function to give sidelobes tapering down beyond a specified number $\bar{n}$. Hansen [4] subsequently concluded that the proper selection of $\bar{n}$ for the Taylor excitations gives the highest AE for a given sidelobe level when compared with other excitations known to him. Also, Kinsey [5] recently noted that, for practical purposes, the selected Taylor excitations are as good as can be achieved.

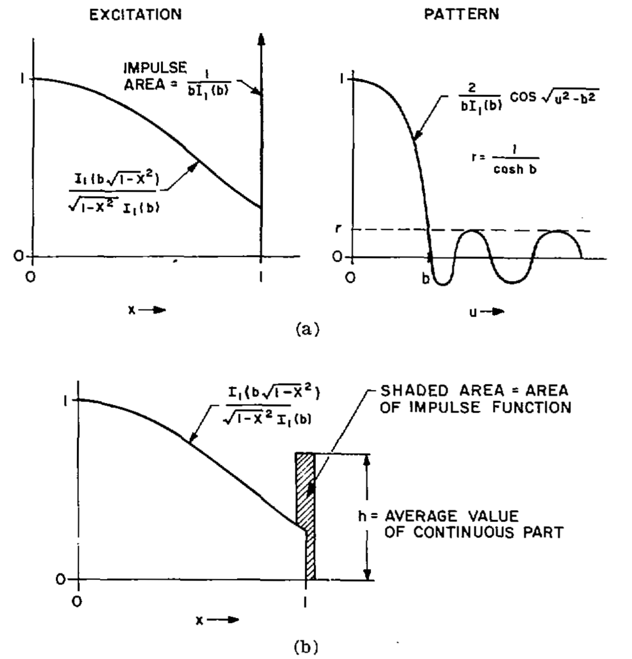

Fig. 1. Excitation and pattern functions for minimum beamwidth and maximum aperture efficiency. (a) van der Maas limit functions. (b) Proposed (Wheeler) excitation.

III. Proposed Excitations

Wheeler, in private discussions with the author and in his notes some years ago [10], has described a different approach to the problem, which builds on the peculiarities of the excitation function. He proposes a unique modification of the van der Maas limit function, which is not restricted to analytic functions everywhere between the edges. Fig. 1(b) shows the basic concept. The area of the impulse at each edge is retained but is redistributed, as shown, so that the following results occur.

1) The sidelobes do not exceed the same level, and the outer sidelobes taper down to take less power.

2) The shape of the main beam is approximately preserved by also retaining the second moment of each edge pulse around the aperture center. This is accomplished by locating the "center of gravity" of the shaded area at the end of the curve. (For a narrow pulse, a number of higher order moments are approximately retained, so the near sidelobes are approximately preserved.)

3) The maximum aperture efficiency is approached by setting the height of the pulse near the average of the curve. (It is reasoned that this pulseheight should approach maximum efficiency, because any departure from the average excitation increases the total power without increasing the useful power density in the beam direction.)

In Fig. 1(b) it is noted that the redistributed area extends outside of the initial aperture. Renormalization of the aperture width results in a beamwidth slightly greater than the van der Maas minimum beamwidth. This is simply the expected tradeoff between the minimum beamwidth and maximum AE, corresponding to the choice of $\bar{n}$ in Taylor's distribution. Fig. 2 shows the proposed family of excitations, normalized to the same aperture width.

Manuscript received November 24, 1969; revised December 9, 1970.

[1] The term "sidelobe level" is used in the accepted sense, meaning the level not to be exceeded by any of the sidelobes. In the Dolph–Chebyshev and van der Maas formulation, this is the level of all sidelobes.

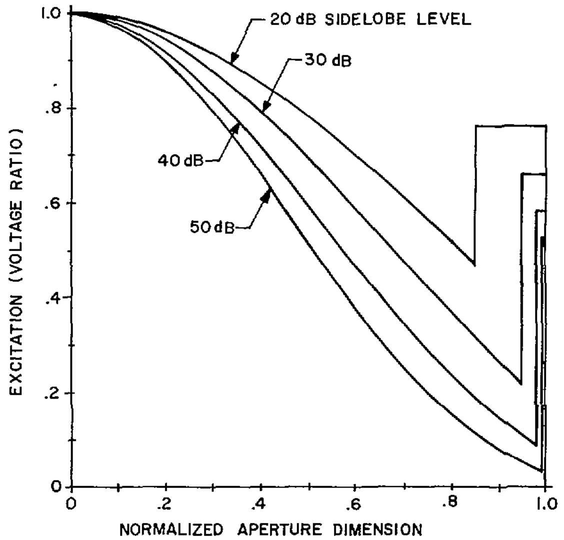

Fig. 2. Proposed excitation family for maximum aperture efficiency with given sidelobe level.

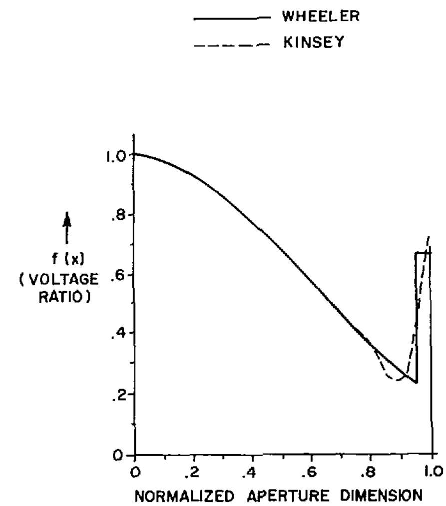

Fig. 3. Comparisons of two forms of excitation intended for maximum aperture efficiency with 30-dB sidelobe level.

TABLE I

CHARACTERISTICS OF VAN DER MAAS LIMIT FUNCTIONS

	r=SIDELOBE LEVEL (VOLTAGE RATIO)					
	SIDELOBE LEVEL -dB OF r					
	15	20	25	30	35	40
	b = anticosh (1/r)					
	2.412	2.993	3.571	4.147	4.723	5.298
x	$f_1(x) = \frac{I_1(b\sqrt{1-x^2})}{\sqrt{1-x^2}\; I_1(b)}$					
0.1	.9940	.9915	.9889	.9862	.9835	.9807
0.2	.9763	.9665	.9562	.9458	.9352	.9247
0.3	.9473	.9257	.9036	.8813	.8591	.8372
0.4	.9077	.8709	.8337	.7969	.7610	.7263
0.5	.8586	.8040	.7500	.6978	.6482	.6015
0.6	.8012	.7275	.6564	.5899	.5288	.4732
0.7	.7369	.6440	.5574	.4794	.4106	.3507
0.8	.6673	.5565	.4574	.3721	.3006	.2417
0.9	.5942	.4679	.3603	.2729	.2043	.1516
1.0	.5190	.3809	.2699	.1859	.1252	.0827
	AVERAGE VALUE OF $f_1(x) = \frac{1/r - 1}{bI_1(b)}$					
	.8250	.7654	.7104	.6620	.6200	.5833
	IMPULSE FUNCTION AREA = $\frac{1}{bI_1(b)}$					
	.17842	.08502	.04234	.02162	.011224	.005893

Table I presents a tabulation of the van der Maas excitation functions and the average value of the continuous part of the excitations, which is the pulseheight in the proposed excitations.

The proposed excitations have been evaluated by numerical methods. The expression defining AE as a ratio of integrals was evaluated as a ratio of sums. The height of the pulse was set to the average value of the curve as given in Table I, and the center of gravity of the pulse was located at the position of the van der Maas impulse. The results of these simple calculations are given in the next section. Additional calculations have shown that a slight advantage is obtained by choosing the pulseheight somewhat below the average and sloping the pulse top downward from the edge of the aperture.

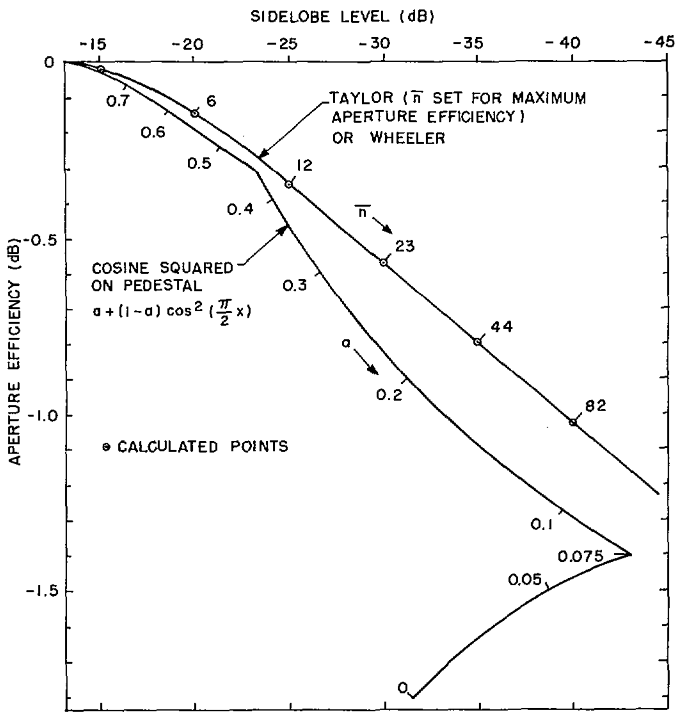

Fig. 4. Aperture efficiency versus sidelobe level for Wheeler, Taylor, and cosine squared on pedestal excitations.

Fig. 3 shows a comparison of the excitation obtained by Kinsey [5] for his highest AE with a 30-dB sidelobe level and the one proposed by Wheeler for the same sidelobe level. The similarity over most of the aperture is remarkable, considering the laborious computations required by one in contrast to the simple computation of the other.

IV. COMPARISON OF EXCITATIONS AND CONCLUSIONS

Fig. 4 presents the values of aperture efficiency calculated for several sidelobe levels for the Wheeler excitations. The values presented are for the simple formula previously prescribed and do not include the slight advantage of a lower pulseheight and tilted top. For comparison, two other well-known excitations are included. The values for the Taylor excitations ($\bar{n}$ set for maximum aperture efficiency) were obtained from [3] and [5]. The "cosine squared on a pedestal" excitations are simple and give efficiency within 6 percent of the maximum for sidelobe levels down to 40 dB [9].

As shown in Fig. 4, the Taylor and Wheeler excitations provide, within the accuracy of the calculations and plotting, equal aperture efficiencies. The practical distinctions between these two functions are in their description and ease of calculation. The $\bar{n}$ values for the Taylor functions had to be determined numerically and have no simple rule for approaching maximum aperture efficiency. The calculation of the Taylor excitation curve involves a summation of $\bar{n} + 1$ terms; the Wheeler excitation curve is given by one term plus the edge pulse.

There remains to be uniquely determined the excitation which will give the maximum aperture efficiency for a given sidelobe level. The formula proposed herein is close enough for practical purposes and is the simplest yet stated. It departs from the artificial restriction of continuity everywhere between the edges, and this freedom may be found essential to the ultimate formulation.

Acknowledgment

The principle underlying this communication was initially described by H. A. Wheeler. The author wishes to express his gratitude to him for his guidance and to the staff of Wheeler Laboratories for their assistance in the performance of this study.

Alfred R. Lopez
Wheeler Labs.
Smithtown, N. Y. 11787

References

[1] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beam width and side-lobe level," *Proc. IRE*, vol. 34, June 1946, pp. 335–348. (Minimum beamwidth for linear array.)

[2] G. J. van der Maas, "A simplified calculation for Dolph–Tchebycheff arrays," *J. Appl. Phys.*, vol. 25, Jan. 1954, pp. 121–124. (Simple calculation of Dolph–Chebyshev array for a large number of elements.)

[3] T. T. Taylor, "Design of line-source antennas for narrow beamwidth and low side lobes," *IRE Trans. Antennas Propagat.*, vol. AP-3, Jan. 1955, pp. 16–28. (Minimum bandwidth for a continuous line aperture.)

[4] R. C. Hansen, "Gain limitations of large antennas," *IRE Trans. Antennas Propagat.*, vol. AP-8, Sept. 1960, pp. 490–495; also, "Correction to 'Gain limitations of large antennas'," ibid, vol. AP-13, Nov. 1965, pp. 997–998. (Concluded Taylor with specified $\bar{n}$, has highest known aperture efficiency.)

[5] R. R. Kinsey, "Line source aperture monopulse excitations," to Air Force Cambridge Res. Lab., Cambridge, Mass., General Electric Co. Rep. AFCRL-68-0254, Apr. 1968.

[6] G. A. Campbell, "The practical application of the Fourier integral," *Bell Syst. Tech. J.*, vol. 7, Oct. 1928, pp. 639–707, (see p. 686, no. 871).

[7] H. Jasik, *Antenna Engineering Handbook*. New York: McGraw-Hill, 1961, pp. 2–22.

[8] L. B. Brown and G. A. Scharp, "Tschebyscheff antenna distribution, beamwidth, and gain tables," Naval Ordnance Rep. 4629 (Rep. 383, Naval Ordnance Lab, Corona, Corona, Calif.), Feb. 1958. (Exitations and aperture efficiencies in close agreement with result presented herein.)

[9] J. F. Ramsay, "Lambda functions describe antenna diffraction patterns," *Microwaves*, June 1967, pp. 69–107.

[10] H. A. Wheeler, unpublished notes, 1958. (The proposal to approach maximum aperture efficiency by substituting for the impulses of van der Maas, a pulse of equal area and of height comparable with average excitation.)

[11] "IEEE standard definitions of terms for antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-17, May 1969, pp. 262–269.

[12] A. R. Lopez, "A line source exitation for maximum aperture efficiency with a given sidelobe level," in *1970 IEEE Int. Symp. Antennas Propagat. Dig.*, pp. 296–302.

Singular Integral Equation Solution for Electrically Small Short Cylindrical Antenna

Abstract—**It is shown, that in the case of the electrically small tubular cylindrical antenna whose length-to-radius ratio is 0(1) or smaller, that the integral equation involved readily reduces to a simple singular equation, the solution of which is in terms of elementary functions.**

Manuscript received January 26, 1970; revised February 22, 1971.

I. Introduction

When Maxwell's equations are applied to the case of a thin walled tubular perfectly conducting cylindrical antenna, one result is an integral equation for the current. It can be shown [1] that the integral equation for the total current on the tube is

$$\int_{-h}^{h} \left\{ \frac{dI(z')}{dz'} \frac{d}{dz} G_0(z - z') + k_0^2 I(z') G_0(z - z') \right\} dz' = F(z), \qquad |z| < h \quad (1)$$

where $2h$ is the antenna length, $k_0 = \omega(\mu_0\epsilon_0)^{1/2}$, and time variation is $\exp(i\omega t)$. $F(z)$ represents the z component of the total applied electric field on the surface of the antenna.

In the past, almost all solutions of (1) have dealt with the situation, $a \ll \lambda$, $h \gg a$. In this case, the logarithmically singular behavior of the kernel $G_0(z)$ as $|z| \to 0$ is not important, unless fine detail of the current is required near the ends of the antenna or at concentrated sources on the surface of the antenna. If the other extreme where $a \gg h$ is considered, however, it is found that the logarithmic singularity is the dominant feature of $G_0(z)$. It is natural, therefore, that the well-developed theory of singular integral equations [2], [3] should be invoked in this case. The result is that when $h \ll a \ll \lambda$, an accurate closed-form solution of the integral equation can be obtained. Equally important, however, is the fact that this result is the principal part of the solution for thick antennas where $a \gg h$ and a/λ is large.

Applications of this work may be found in the design of small field probes where accurate and simple solutions are required; in the analysis of thick antennas; in the determination of the electric field close to edges and feed apertures to facilitate studies of microwave breakdown, and in graduate teaching courses where it is desired to display the nature of the current distribution at edges and sources in a simple and elegant manner. One case where the theory has been applied successfully is in the design of a miniature probe for measuring differential and transient soil moisture content.

Thus this communication introduces singular integral equation theory to the solution of the cylindrical antenna problem for those cases where the antenna length is short compared with its radius.

II. Solution for Concentrated Source

Let the antenna excitation be defined by

$$E_z(a + 0, z) = -\delta(z), \qquad |z| < h \quad (2a)$$

$$E_z(a - 0, z) = 0, \qquad |z| < h. \quad (2b)$$

with the corresponding physical model as shown in Fig. 1(a) and the mathematical abstraction as shown in Fig. 1(b). Note that the delta-function excitation has to be understood as part of a mathematical process whereby the finite aperture problem may be solved. Now the right-hand side of (1) becomes

$$F(z) = -\frac{2\pi i \delta(z)}{Z_0} + \frac{2\pi i k_0 a}{Z_0} \left.\frac{\partial G_0}{\partial r'}\right|_{r'=a} \quad (3)$$

Making the substitutions

$$k_0 h = H \quad (4a)$$

$$k_0 a = A \quad (4b)$$

$$z = ht \quad (4c)$$

$$\frac{h}{8a} = R \quad (4d)$$

$$\phi(t) = \frac{dI(t)}{dt} \quad (4e)$$