Cellular Telecommunications: Estimating Shadowing Effects Using Wedge Diffraction

Alfred R. Lopez
GEC-Marconi Hazeltine
Mail Stop 1-83
450 Pulaski Road
Greenlawn, NY 11740-1606
Tel: (516) 262-8021
Fax: (516) 262-8053
E-mail: lopeza@hazeltine.com

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1. Abstract

Knife-edge diffraction has been used to estimate the effects of blocking obstacles and terrain for cellular telecommunications. Wedge diffraction is more appropriate in many cases. This paper presents simple formulas that quantify the effects of shadowing caused by rising terrain using wedge diffraction. The shadowing effect is directly related to the typical cellular-telecommunications case, where an inverse fourth-power relationship of received power with distance is characteristic.

2. Introduction

In most cellular-telecommunications applications, except near the base station, the transmission loss over a plane earth (flat ground) is characterized by an inverse fourth-power ($1/R^4$) relationship of received power with distance from the base-station antenna. For situations where an obstacle blocks the line of sight, knife-edge-diffraction theory has been used to estimate the amount of signal attenuation caused by the blocking obstacle [1, 2]. For many cases (see Figure 1) wedge-diffraction theory is more appropriate than knife-edge-diffraction theory, and provides estimates of shadowing effects directly related to the $1/R^4$ flat-ground case. In [3], a simple formula was presented that determines the wedge shadowing loss with respect to the flat-ground case, with the receiver located in the deep-shadow region. This article extends those results to the case where the receiver is near and away from the shadow boundary.

3. Flat-ground propagation

Ground-to-ground propagation is generally characterized by a transmitting antenna and its negative image. The flat-ground geometry is shown in Figure 2. The equation for received power is

$$P = EIRP \times G \times \left(\frac{\lambda}{4\pi R}\right)^2 \times 4\sin^2\left[\frac{2\pi}{\lambda} H_T \sin(\theta)\right] \qquad (1)$$

where P is the received power, $EIRP$ is the effective isotropic radiated power, G is the receiving-antenna gain, $(\lambda/4\pi R)^2$ is the free-space propagation factor, and the last term is the two-element array factor. If $\sin(\theta) \approx \frac{H_R}{R}$, and if $2\pi \frac{H_T}{\lambda}\frac{H_R}{R} \le \frac{\pi}{16}$, then

$$P = EIRP \times G \times \frac{{H_T}^2 {H_R}^2}{R^4}. \qquad (2)$$

Note the $1/R^4$ propagation factor in this equation. For comparison to a wedge, it is helpful to define flat ground as a wedge, as shown in Figure 3.

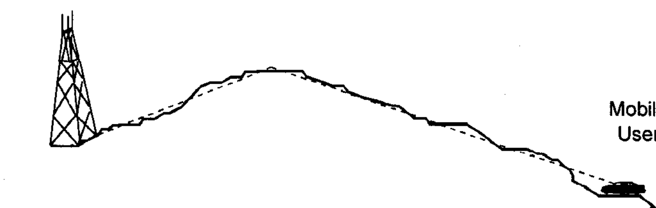

Figure 1. Rising and falling terrain, approximated by a wedge.

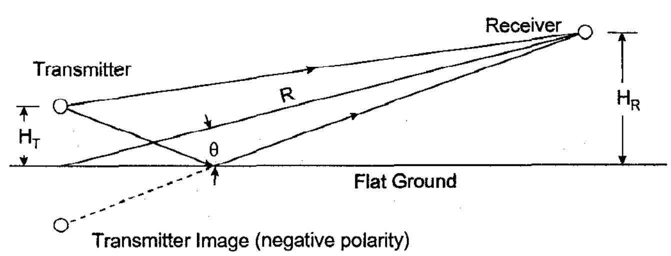

Figure 2. The flat-ground propagation geometry.

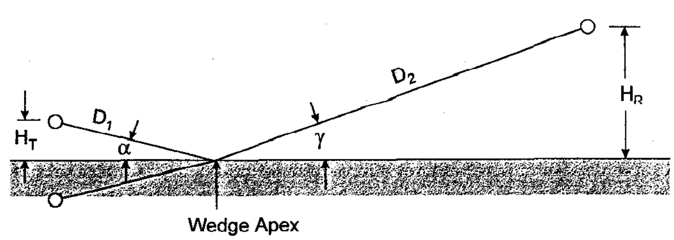

Figure 3. The flat-ground wedge geometry.

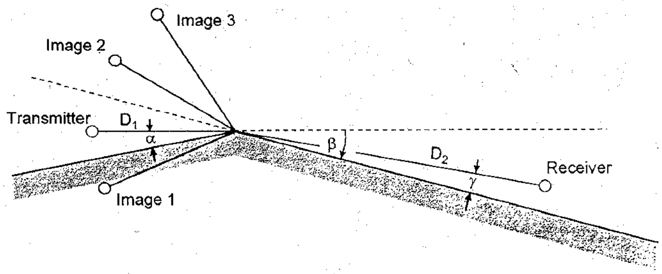

Figure 4. The wedge geometry.

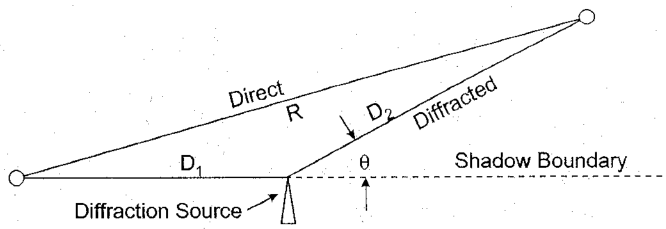

Figure 5. The geometry for the shadow-boundary approximation.

The condition for $1/R^4$ propagation is converted to

$$2\pi \frac{H_T}{\lambda} \frac{H_R}{R} = 2\pi \frac{\alpha D_1 \gamma D_2}{\lambda (D_1 + D_2)} = 2\pi \alpha \gamma \frac{D}{\lambda} \leq \frac{\pi}{16}, \quad (3)$$

where

$$D = \frac{D_1 D_2}{D_1 + D_2}.$$

For comparison of a wedge to flat ground, γ is arbitrarily set equal to

$$\boxed{\gamma = \frac{\lambda}{32 \alpha D}}.$$

This corresponds to a receiver being located at a point near the ground that is down 14.2 dB from the first peak of the elevation-lobe pattern.

4. Wedge diffraction

With respect to flat ground, the wedge introduces two new transmitter images that are created by the wedge surface near the receiver, as shown in Figure 4. Diffracted signals emanate from the apex of the wedge, and provide radiated signals in the shadow region.

4.1 Deep-shadow approximation

For the following conditions,

$$\alpha_{Wedge} = \alpha_{Flat\ Ground},$$

$$\gamma_{Wedge} = \gamma_{Flat\ Ground},$$

$$D_{Wedge} = D_{Flat\ Ground},$$

$1/R^4$ propagation conditions exist, and the receiver is in the deep-shadow region. The ratio of the received wedge power, relative to the received flat-ground power, is given by [3] (see the Appendix)

$$\frac{P_{Wedge}}{P_{Flat\ Ground}} \approx \frac{1}{\pi^4} \left(\frac{\sqrt{\lambda/D}}{\alpha + \beta} \right)^6 = \frac{1}{\pi^4} \frac{1}{\psi^6}, \quad (4)$$

where ψ is the normalized wedge angle:

$$\boxed{\psi = \frac{\alpha + \beta}{\sqrt{\lambda/D}}}.$$

4.2 Approximation for region within ±30° of shadow boundary

The field components are inversely proportional to the path from the source to the observation point. A path-difference factor accounts for the relative amplitude of the field components associated with the path difference. An approximation for the path-difference factor is derived, which allows the computation of signal level versus ψ for a region of ±30° about the shadow boundary. The relevant geometry for this derivation is shown in Figure 5. Approximations for the path-difference factor for the diffracted signal relative to the direct signal, and for the phase difference of the diffracted signal with respect to the direct signal, are derived below.

$$R = \sqrt{D_1^2 + D_2^2 + 2 D_1 D_2 \cos(\theta)},$$

$$R = (D_1 + D_2) \sqrt{1 - \frac{4 D_1 D_2}{(D_1 + D_2)^2} \sin^2\left(\frac{\theta}{2}\right)}.$$

The path-difference factor (voltage ratio) is given by

$$\frac{R}{D_1 + D_2} = \sqrt{1 - \frac{4 D_1 D_2}{(D_1 + D_2)^2} \sin^2\left(\frac{\theta}{2}\right)}.$$

If $|\theta \leq 30°|$, and since $\frac{4 D_1 D_2}{(D_1 + D_2)^2} \leq 1$, then

$$0.966 \leq \frac{R}{D_1 + D_2} \leq 1,$$

and

$$\boxed{\frac{R}{D_1 + D_2} \approx 1}.$$

The phase difference is given by

$$\Delta\phi = \frac{2\pi}{\lambda} (D_1 + D_2 - R),$$

$$R \approx D_1 + D_2 - 2D \sin^2\left(\frac{\theta}{2}\right),$$

$$\boxed{\Delta\phi \approx \pi \frac{D}{\lambda} \theta^2}.$$

With the above approximations, it is now possible to compute the total signal versus ψ, the normalized wedge angle. The total field for a point source and for its diffracted component, normalized to the unperturbed field for the point source at the shadow boundary and at a distance of $D_1 + D_2$ are given by

$$E_{Total} = \frac{D_1 + D_2}{R} u(\theta)\exp(j\Delta\phi) + E_{Diffracted}(\theta)$$

$$\approx u(\theta)\exp(j\Delta\phi) + E_{Diffracted}(\theta),$$

where

$$u(\theta) = 1 \text{ if } \theta \geq 0,$$

$$u(\theta) = 0 \text{ if } \theta < 0.$$

The total signal for the wedge consists of four basic components:

1. direct from the transmitter;
2. reflected by the wedge surface near the transmitter;
3. reflected by the wedge surface near the receiver;
4. doubly reflected by the wedge surfaces near the transmitter and the receiver.

Each basic component consists of two parts:

A. The main electric-field vector, which is characterized by a unit step function:

$$E_{Main}(\theta) = u(\theta)\exp\left(j\pi D\theta^2/\lambda\right); \quad (5)$$

B. The diffracted electric-field vector, which is characterized by a sgn function:

$$E_{Diffracted}(\theta) = \text{sgn}(\theta)F(\theta), \quad (6)$$

where

$$\text{sgn}(\theta) = 1 \text{ if } \theta \geq 0,$$

$$\text{sgn}(\theta) = -1 \text{ if } \theta < 0,$$

$$F(\theta) = \left[\frac{\tanh(v)}{2v} - \frac{ve^{-1.5v}}{4}\right]\exp\left[j\frac{\pi}{4}\tanh(v/2.4)\right],$$

$$v = 2\pi\sqrt{D/\lambda}\left|\sin(\theta/2)\right|.$$

The sign and values of θ for the main and diffracted parts are given in Table 1.

The ratio of received powers for a wedge and for flat ground are shown in Figures 6 and 7. Each trace is developed by setting α and D/λ to the specified values, and varying β over a given range. In Figure 6, the range of ψ extends over negative and positive values. For negative values of ψ, the wedge is concave, and signal gain is possible. For positive values of ψ, the wedge is convex, and signal loss is a characteristic. The dashed curve corresponds to the relationship first presented in [3]. It is a good approximation for the deep-shadow region.

For cellular telecommunication, loss is a principal concern. Figure 7 presents the wedge loss factor over a range of α, D, and ψ. This range of parameters should encompass nearly all situations of interest for cellular telecommunications. The range of the vertical scale is 60 dB, and the range for ψ is 10 for $D/\lambda = 1000$, 20 for $D/\lambda = 5000$ and 10000, and 60 for $D/\lambda = 100000$. The value for ψ such that the receiver is at the shadow boundary is given by

$$\psi_{Shadow} = \frac{\alpha + \gamma}{\sqrt{\lambda/D}} = \frac{\alpha + \dfrac{\lambda}{32\alpha D}}{\sqrt{\lambda/D}}$$

$$= \alpha\sqrt{\frac{D}{\lambda}} + \frac{1}{32\alpha}\sqrt{\frac{\lambda}{D}}.$$

For the range of parameters given in Figure 7, $\psi_{Shadow} \approx \alpha\sqrt{D/\lambda}$. Some irregularities in the traces shown in Figure 7 are related to the shadow-boundary transition.

Table 1. The sign and values of θ for the main and diffracted components.
Note: $\gamma = \lambda/(32\alpha D)$

Component	Sign of Main Part	Sign of Diffracted Part	Value of θ
1. Direct from transmitter	+	−	$\gamma - \beta$
2. Reflected by wedge surface near the transmitter	−	+	$\gamma - \beta - 2\alpha$
3. Reflected by wedge surface near the receiver	−	+	$-\gamma - \beta$
4. Doubly reflected by wedge surfaces near the transmitter and the receiver	+	−	$-\gamma - \beta - 2\alpha$

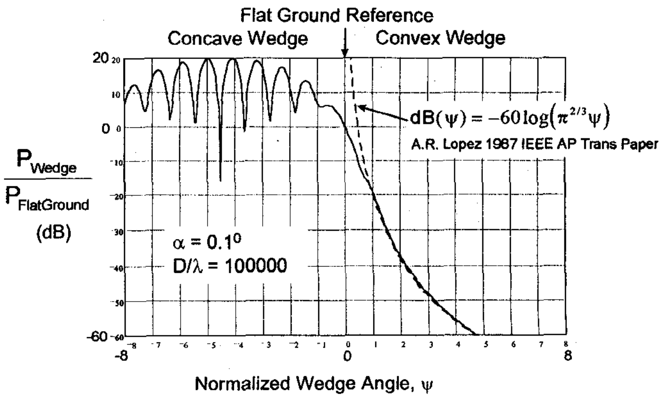

Figure 6. The wedge factor relative to flat ground, for negative and positive values of ψ.

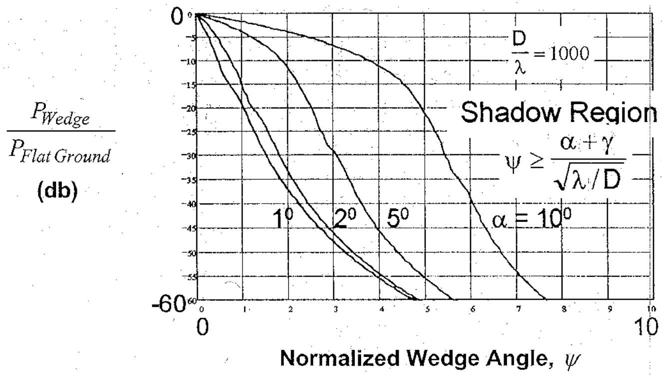

Figure 7a. The wedge factor relative to flat ground as a function of the normalized wedge angle, ψ, with α as a parameter, for $D/\lambda = 1000$.

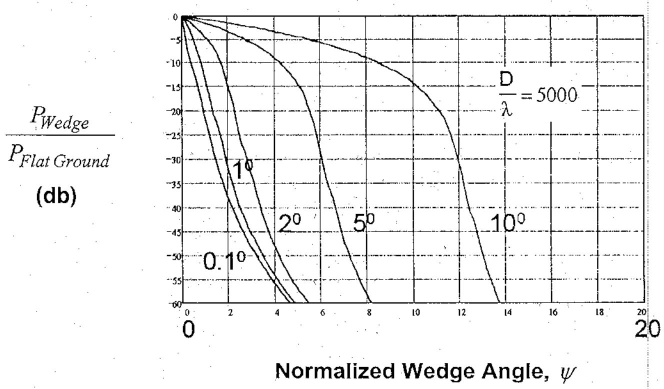

Figure 7b. The wedge factor relative to flat ground as a function of the normalized wedge angle, ψ, with α as a parameter, for $D/\lambda = 5000$.

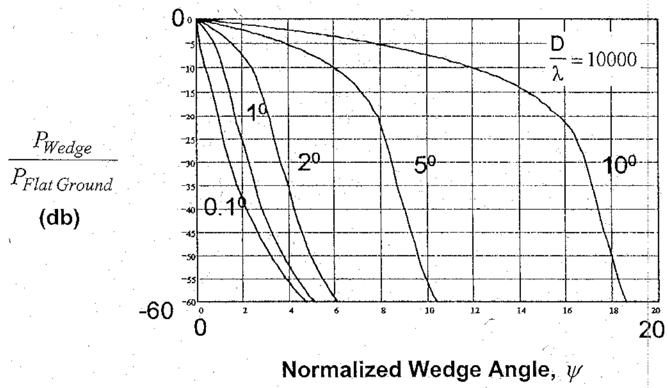

Figure 7c. The wedge factor relative to flat ground as a function of the normalized wedge angle, ψ, with α as a parameter, for $D/\lambda = 10000$.

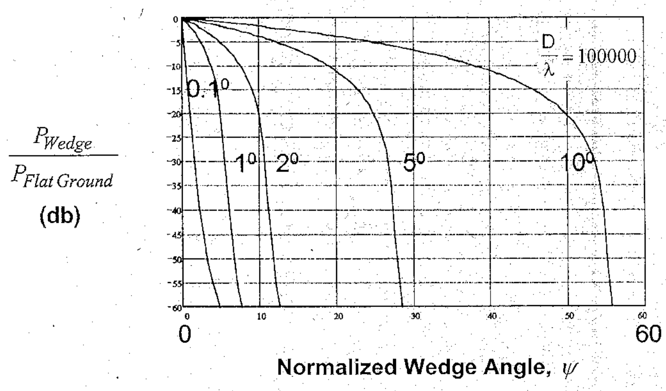

Figure 7d. The wedge factor relative to flat ground as a function of the normalized wedge angle, ψ, with α as a parameter, for $D/\lambda = 100000$.

5. Conclusion

Wedge diffraction has been shown to be helpful in estimating the effects of shadowing caused by rising and falling terrain between a cellular-base-station antenna and a user. It provides estimates relative to the flat-ground case, which has an inverse fourth-power dependence of the received power with distance. The results are generally applicable to any situation where a transmitter and receiver are located close to the ground.

6. References

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7. Appendix

This presents a derivation of

$$\left|\frac{E_{Wedge}}{E_{Flat\,Ground}}\right| \approx \frac{1}{\pi^2}\left(\frac{\sqrt{\lambda/D}}{\alpha+\beta}\right)^3.$$

Wedge component (relative to the free-space component at a distance of $D_1 + D_2$): The following is derived with reference to Figure 4 and Equation (6). If $\nu \geq 2$, $\sin(\theta/2) \approx \theta/2$, and the receiver is in the deep-shadow region, then

$$\left|E_{Diffracted}\right| \approx$$

$$\frac{1}{2\pi\sqrt{D/\lambda}}\left(\frac{1}{\gamma-\beta} - \frac{1}{\gamma-\beta-2\alpha} - \frac{1}{-\gamma-\beta} + \frac{1}{-\gamma-\beta-2\alpha}\right),$$

$$\left|E_{Diffracted}\right| \approx$$

$$\frac{1}{2\pi\sqrt{D/\lambda}}\left(\frac{-1}{\gamma+\alpha-\varphi}+\frac{1}{\gamma-\alpha-\varphi}+\frac{1}{-\gamma+\alpha-\varphi}-\frac{1}{-\gamma-\alpha-\varphi}\right),$$

where

$$\varphi = \alpha + \beta,$$

$$\left|E_{Diffracted}\right| \approx \frac{2\varphi}{2\pi\sqrt{D/\lambda}}\left[\frac{1}{(\gamma+\alpha)^2-\varphi^2}-\frac{1}{(\gamma-\alpha)^2-\varphi^2}\right],$$

$$\left|E_{Diffracted}\right| \approx \frac{2\varphi}{2\pi\sqrt{D/\lambda}}\left\{\frac{4\alpha\gamma}{\left[(\gamma+\alpha)^2-\varphi^2\right]\left[(\gamma-\alpha)^2-\varphi^2\right]}\right\}.$$

If α and γ are small with respect to φ (if the transmitter and receiver are near the ground), then

$$\left|E_{Diffracted}\right| \approx \frac{4\alpha\gamma}{\pi\sqrt{D/\lambda}}\left(\frac{1}{\varphi^3}\right).$$

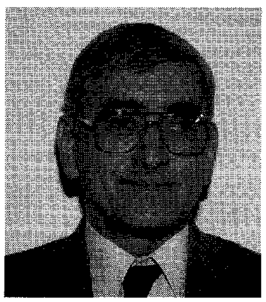
Flat-ground component (relative to the free-space component at distance of $D_1 + D_2$):

$$\left|E_{Flat\,Ground}\right| \approx 4\pi\frac{H_T}{\lambda}\frac{H_R}{D_1+D_2} = 4\pi\alpha\gamma\frac{D}{\lambda}.$$

Wedge-to-flat-ground ratio:

$$\left|\frac{E_{Diffracted}}{E_{Flat\,Ground}}\right| \approx \frac{1}{\pi^2}\left(\frac{\sqrt{\lambda/D}}{\alpha+\beta}\right)^3 = \frac{1}{\pi^2}\frac{1}{\psi^3}.$$

Introducing Feature Article Author



Alfred R. Lopez is a Life Fellow of the IEEE. He received a BEE from Manhattan College, in 1958, and an MSEE from the Polytechnic Institute of Brooklyn, in 1963. He is a Principal Engineer at GEC-Marconi Hazeltine. He started his career at Wheeler Laboratory, in 1958, as an antenna-design specialist. He has made contributions to the theory and practice of electronically scanned antennas. He is currently involved with the development of intelligent antennas for cellular telecommunications.

Mr. Lopez has been awarded 23 US Patents, and has published several papers in the IEEE *Transactions*. He was the recipient of the IEEE Antennas and Propagation Society's Harold A. Wheeler Award in 1988, the IEEE Region 1 Award in 1990, and the IEEE Long Island Section Harold A. Wheeler Award in 1993.

In Memoriam: Ralph Kleinman

The scientific community lost a dear friend and colleague when Ralph E. Kleinman died on February 23, 1998, in Newark, Delaware, after suffering a stroke. Ralph was UNIDEL Professor of Mathematical Sciences at the University of Delaware, and Director of the Center for the Mathematics of Waves, which he established in 1988.

Ralph Kleinman was born in 1929, and grew up in Lindenhurst and Forest Hills, New York. He received a BA in mathematics from New York University, in 1950, and an MA in mathematics from the University of Michigan, in 1951. He worked at Michigan's Radiation Laboratory for two years before serving in the US Army at Redstone Arsenal, in Alabama. He returned to the Radiation Lab in 1955. While there, he received a Fullbright fellowship to study at the Delft University of Technology, where he obtained a Doctor in de Technische Wetenschappen (equivalent to a US PhD) in 1961.

Until 1968, when he moved to Delaware, Ralph was a research mathematician at the Radiation Lab of the University of Michigan. In his early work, he concentrated on radiation and scattering, providing rigorous foundations for computational work being carried out at the lab. He was attracted to low-frequency scattering early in his career, and returned to the subject often. At the time of his death, he had just completed a volume on low-frequency scattering with George Dassios, of the University of Patras.