MEMO: File

FROM: H. A. Wheeler

SUBJECT: Circuit Representation of the radiation from a disc-loaded monopole.

[1] H. A. Wheeler, "Small antennas", IEEE Trans., vol. AP-23, pp. 462-469; Jul. 1975.
[2] -----, "The matching area for a small antenna", IEEE Trans., vol. AP-30, pp. , 1982.
[3] -----, "A simple formula for the capacitance of a disc on dielectric on a plane", IEEE Trans., vol. MTT-30, pp. , 1982.
[4] H. Levine, "Theory of the circular diffraction antenna", Jour. Appl. Phys., vol. 22, pp. 29-43; Jan. 1951.
[5] N. Marcuvitz, "Waveguide Handbook", McGraw-Hill, Rad. Lab. Ser., vol. 10; 1951. (Coaxial line radiating into semi-infinite space, pp. 213-216.)

Abstract. If a disc-loaded monopole has a radius comparable with the height or greater, and comparable with the radianlength, its behavior is more complicated than that of a "small antenna". The radiation loading outside the cylindrical space under the disc can be approximated by a simple CLR network with constant coefficients determined by the dimensions.

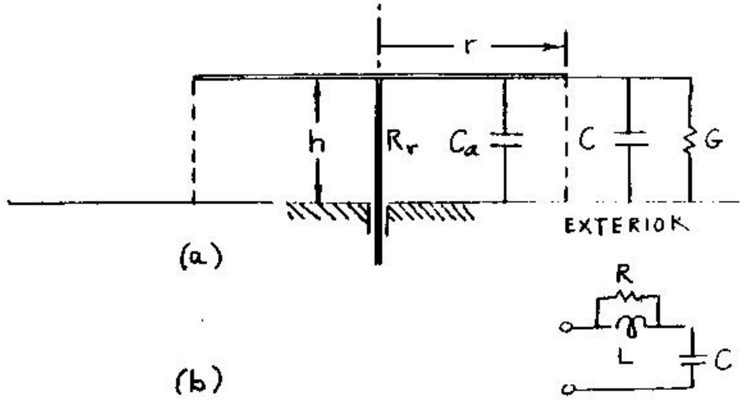

Fig. 1 - Disc-loaded monopole and its representation.

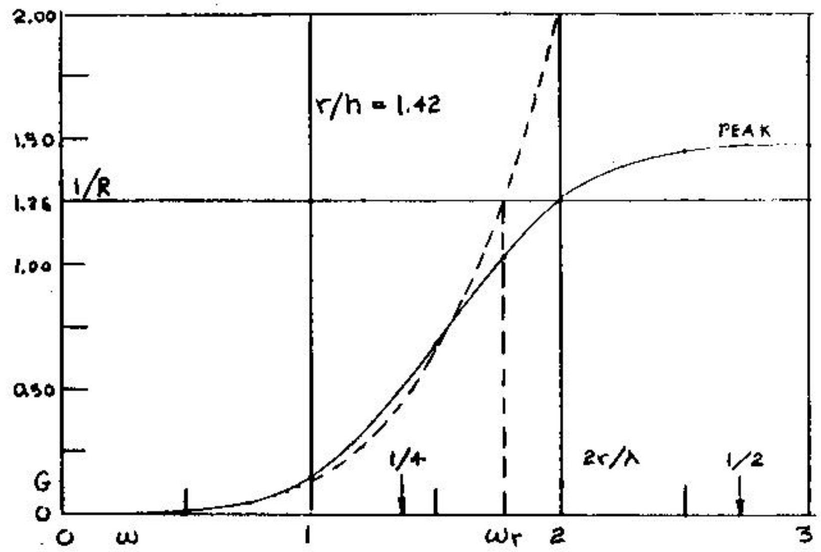

Fig. 2 - Radiation conductance.

The disc-loaded monopole is a simple type of antenna which may be useful as a radiator occupying a cylindrical space above a ground plane. In the limit of a "small antenna", its behavior is simple and easily described. [1][2][3] If the radius is comparable with the radianlengths, a more advanced viewpoint is needed. In particular, the cylindrical boundary under the rim of the disc may be taken as the interface between the exterior field of radiation and the interior field associated with the connecting circuits. The primary purpose here is the representation of the exterior loading in terms of a simple CLR network with constant coefficients.

Symbols.

r = radius of thin disc.

h = height of disc above ground plane.

C_a = interior capacitance (under the area of the disc).

C = exterior capacitance (all except C_a).

r' = "effective radius" for exterior C.

r'' = "effective radius" for total C.

G = frequency-dependent shunt conductance of radiation loading.

L = constant inductance in representation.

R = constant resistance in representation.

R_r = frequency-dependent radiation resistance as seen in a vertical wire at the axis of the disc.

λ = free-space wavelength.

$\lambda/2\pi$ = radianlength.

$\omega = 2\pi f$ = radianfrequency.

λ_1, ω_1 = values at low-frequency cutoff.

λ_r, ω_r = values at resonance of CL.

ϵ_o = ($1000/36\pi$ = 8.84 pF/m) = electrivity of free space.

μ_o = (0.4π = 1.257 µH/m) = magnetivity of free space.

R_o = (120π = 377 ohms) = radiation resistance of square area.

sub-1 = reference values for normalizing.

Fig. 1(a) shows the disc dimensions and its interior and exterior properties. Fig. 1(b) shows the proposed representation of its exterior properties.

In the limit of low frequency, Fig. 1(a) is evaluated. From [3]:

$$C_a = \epsilon_o \pi r^2/h \qquad (1)$$

$$C_a + C = \epsilon_o r \left[\pi r/h + 8 + \frac{2}{3} \ln \frac{1 + 0.8(r/h)^2 + (0.31\, r/h)^4}{1 + 0.9(r/h)} \right] \qquad (2)$$

Express in terms of r' and r";

$$C = \epsilon_o\, 8\, r' \qquad (3)$$

$$r'/r = 1 + \frac{1}{12} \ln \frac{1 + 0.8(r/h)^2 + (0.31\, r/h)^4}{1 + 0.9(r/h)} \qquad (4)$$

$$C_a + C = \epsilon_o\, 8\, r'' \qquad (5)$$

$$r''/r = \frac{\pi}{8} r/h + r'/r \qquad (6)$$

$$r''/r' = 1 + C_a/C \qquad (7)$$

The ln term is a minor fraction which may be ignored for estimating purposes. The radiation resistance, referred to the current in the axial wire, is

$$R_r = \frac{1}{3\pi} R_o (2\pi h/\lambda)^2 \qquad (8)$$

The effective height (h) is clearly defined as the actual height of the thin disc, ignoring the extra capacitance of the fine wire. This frequency-dependent R_r in series with the total capacitance ($C_a + C$) is transformed to an equivalent frequency-dependent parallel conductance:

$$G = \omega^2 (C_a + C)^2 R_r = (1/R_o) \frac{64}{3\pi} (2\pi r''/\lambda)^2 (2\pi h/\lambda)^2 \qquad (9)$$

Note that $R_o = \sqrt{\mu_o/\epsilon_o}$; $\omega = 2\pi/\lambda\sqrt{\epsilon_o \mu_o}$; $\lambda/2\pi = 1/\omega\sqrt{\epsilon_o \mu_o}$.
This G has the expected (frequency)^4 variation.

In the limit of high frequency, the aperture loading approaches the "plane-wave" radiation R of the cylindrical interface:

$$R = R_o h/2\pi r \quad (10)$$

This extreme is represented in Fig. 1(b) by an elementary high-pass filter terminated in R.

The second-order low-frequency behavior of the exterior loading is obtained in Fig. 1(b) by choice of a constant L across R. The equivalent parallel G from (9) is obtained by:

$$G = (\omega C)^2(\omega L)^2/R \;;\; L = \sqrt{GR}/\omega^2 C \quad (11)$$

In terms of dimensions:

$$L = \mu_o h(\sqrt{h/r})(r''/r')(1/\pi\sqrt{6}) \quad (12)$$

The wavelength of CL resonance is:

$$\lambda_r = 2\pi h(r/h)^{1/4}\sqrt{r''/r}\sqrt{8/\pi\sqrt{6}} \quad (13)$$

In words, the radianlength of resonance is slightly greater than the height. The G is maximum at a lesser radianlength.

Fig. 2 shows the frequency variation of G. It is based on the example to be described. It is typical of the range of shape and size for which the above formulation is intended. The dashed curve is a continuation of the a^4 variation from the limiting case of low frequency ("small antenna").

The G variation is primarily dependent on the radius, so the points of quarter-wave and half-wave diameter are marked. The latter is near the peak of G, which is wide. The G approaches a limit for small height. On the other hand, the parallel susceptance has a weak inverse variation with height.

There have been found, no graphs of the radiation loading for the subject configuration. The nearest is the annular slot in a plane, radiating into half-space. [4][5] Those graphs show similar principal features. The G for a narrow slot should be equal to that for a disc at small height, but the reference graphs do not go to that extreme. They give some assurance of the qualitative validity of the subject representation, and hence a fair approximation for practical purposes.

The usefulness of the subject representation is further assured by the fact of its close approximation in the low-frequency region, where the small-antenna limitations are most severe. [2] For a bandwidth ratio of 1:2 or 1:3 from unit ω in Fig. 2, most of the matching area is crowded near the lower cutoff, and that is where the approximation is most reliable. That is where the representation has least departure from the "small-antenna" model yielding the dashed curve.

A simple description of the graph in Fig. 2 could be based on the regions below and above the frequency ω_r of CL resonance:

- At lower frequencies, the ω^4 dashed curve.
- At higher frequencies, the 1/R level.

As an example, take the shape and size of a model reported by Goubau, but enlarged 3/2 for a low cutoff at 300 MHz instead of 450. It is described here in physical units and in normalized units for convenience of computation (and graph in Fig. 2).

	r	0.092 m	
	h	0.065	
	r/h	1.415	1.415
(2)	$(C_a + C)$	10.13 pF	1.
(1)	C_a	3.62	0.357
	C	6.51	0.643
	ω_1	2π0.3 GHz	1.
	λ_1	1. m	1.
	C_1	10.13 pF	1.
	L_1	0.0278 μh	1.
	R_1	0.0524 K.ohm	1.

(4)	r'/r	1.012	1.012
(6)	r''/r	1.568	1.568
(8)	R_r/ω^2(lo-f)	0.00188	0.1274
(9)	G/ω^4(lo-f)	0.193	0.1274
(10)	R	0.0424 K.ohm	0.809
(11)(12)	L	0.0139 μH	0.500
(13)	λ_r	0.567	
	ω_r	3.324	1.764

The first column is based on a consistent set of units (GHz, K.ohm, m.mho, pF, μH). Where unit value appears in the second column, the corresponding value in the first column is the reference for normalizing. The formulas are valid for the set of values in either column.

The nominal value of radiation PF at low cutoff is 0.1274, which appears in the second column. It is based on total (C_a + C) so it is not relevant to the representation of the external radiation loading.

This example can be matched to a 50-ohm line within the following VSWR, based on a computed double-tuned network that appears to be realizable:

BW ratio 1:2	VSWR < 2.73
1:3	3.12

The radiation loading has a peculiarity that is relevant to the rationale for its network representation. Toward the limit of a small height:

- The radiation G is independent of height;
- The shunt susceptance, proportional to C, is weakly dependent on the height.

Therefore the admittance departs from the rule of "minimum susceptance" determined by the conductance G.

The representation in Fig. 1(b) is a "minimum-susceptance" network, and to that extent is inconsistent with this concept of the radiation loading. A further refinement might be, the relocation of part of C to a direct shunt at the terminals. A rationale for this partition could be conjectured from the formula for C, the second and third of the three parts in (2). The third part (ln...) is directly identified with the fringing field, which is most dense near the rim of the disc and has a weak inverse variation with height. Therefore this part of C might be diverted to a shunt connection at the terminals of the network. Then L would be revalued to retain the same G in the limit of low frequency. This option may be explored later. Its effect would be trivial in the example described, but would be expected to yield a closer representation for much greater r/h.