computer memory, and the methods of Chapter 2. This step is necessary because the residual noise in each cell is modified after the removal of dominant point-source interference in Approach $B_{\text{without baseband threshold detection}}$.

b. Estimate the underlying PDF of each subdivision utilizing the Ozturk algorithm (Chapter 4) and spherically invariant random vectors (SIRVs) (Chapter 12). spherically invariant random vectors are applicable to the case of spatially correlated stochastic data samples (as in a radar that processes $N > 1$ pulses at a time).

Approach C_{residual}, performed after Approach A_{residual} has been performed, consists of the following step:

a. For each residual homogeneous subdivision of the surveillance volume, determine the receiver the detection algorithm of which is best matched to the estimated "noise" voltage spatial PDF of that subdivision. For small-amplitude signal detection, utilize the Middleton locally optimum detector (LOD) by matching its detection algorithm and threshold to the estimated "noise" PDF of each subdivision (Chapter 13). For large-amplitude signal detection, utilize an amplitude-dependent locally optimum detector (after P. Chakravarthi in Chapter 13).

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Rebuttal to "Correct Impedance-Matching Limitations"

Keywords: Impedance matching; Q factor; tuning; Fano; Bode

Hansen [1] implied that Lopez had gross errors in his articles [2, 3]. This is simply not true. Dr. Hansen appears to confuse the fundamental relationships of the Fano equations as they apply to the Q-bandwidth product with his use of his bandwidth improvement factor (BWIF). Fano's equations, presented below, allow computation of the maximum possible bandwidth, B, as limited by:

1. The antenna Q,

2. The maximum permissible reflection coefficient magnitude, Γ, and

3. The complexity of the impedance-matching network as measured by the index n, the number of tuned circuits in the impedance-matching network:

$$B_n(\Gamma) = \frac{1}{Q} F_n(\Gamma).$$

$F_n(\Gamma)$ is a function obtained from Fano's set of three transcendental simultaneous equations that relate QB_n, Γ, and n.

Hansen defined a bandwidth improvement factor, $BWIF$, which is a ratio of bandwidth ratios:

$$BWIF_n = \frac{B_n}{B_1}.$$

In [1], he incorrectly assumed $BWIF_n = QB_n$. This is not valid:

$$BWIF_n \neq QB_n.$$

The discussion below starts with the definition of terms, and then comments on specific statements presented in [1]. These statements are italicized and presented below Hansen's [1] section titles.

1. Definitions

[The definitions are given in Table 1.]

2. Exact Fano Results

"In classic papers circa 1950, Fano developed equations for the bandwidth improvement factor provided by N lossless matching sections."

Fano did not develop equations for the Hansen-defined bandwidth improvement factor. He developed equations that relate QB to Γ and n. Hansen [9] defined one of many possible bandwidth improvement factors. Fano's equations can be used to evaluate Hansen's $BWIF$ and any of the other possible $BWIF$s.

The Fano equations [2-5] are

$$QB = \frac{1}{\delta} = \frac{2\omega_c}{A_1^\infty} = \frac{2\sin\left(\frac{\pi}{2n}\right)}{\sinh(a) - \sinh(b)}, \tag{1}$$

Fano's [2] Equation (36) (Tanner [6] was the first to equate QB to Fano's $\frac{2\omega_c}{A_1^\infty}$: see Appendix 1);

$$\frac{\tanh(na)}{\cosh(a)} = \frac{\tanh(nb)}{\cosh(b)}, \tag{2}$$

Fano's [2] Equation (37);

$$\Gamma = \frac{\cosh(nb)}{\cosh(na)}, \tag{3}$$

Fano's [2] Equation (38). These three transcendental simultaneous equations define the relationship between QB and Γ for a given n.

3. Bad and Good Lopez Results

"Lopez [2, 3] attempted to obtain a simple formula for the bandwidth-improvement factor."

This statement is not correct: Lopez obtained a simple formula for the fundamental relationship among QB, Γ, and n, based on the Fano equations. Hansen's incorrect Equation (1) in [1], attributed to Lopez,

$$BWIF = \frac{1}{b_n \sinh\left[\frac{1}{a_n}\ln\left(\frac{1}{\Gamma}\right)\right] + \frac{1-b_n}{a_n}\ln\left(\frac{1}{\Gamma}\right)}, \tag{4}$$

Table 1. Definitions of terms, and comments.

Symbol	Name	Comment
Q	Antenna Q	The ratio of 2π times the energy stored to the energy radiated and dissipated per cycle [12]
n		Index for the number of impedance matching tuning sections
Γ	Reflection magnitude	Maximum permissible reflection coefficient magnitude within a given bandwidth
B B_n $B_n(\Gamma)$	Matching bandwidth ratio (In this article bandwidth and matching bandwidth are synonymous with matching bandwidth ratio)	Frequency bandwidth ratio over which Γ is not exceeded for an impedance matching circuit with n tuning sections. The matching bandwidth depends on n, Q, and Γ. $B = \frac{f_{High} - f_{Low}}{\sqrt{f_{High} f_{Low}}}$
QB QB_n $QB_n(\Gamma)$	Product of Q and B	Product of Q and the matching bandwidth ratio
δ δ_n $\delta_n(\Gamma)$	Decrement	Hansen [5] defines "δ is decrement (δ is $1/Q$ of the load at band edges)" Hansen fails to state that $\delta_n = 1/QB_n$
$BWIF$ $BWIF_n$ $BWIF_n(\Gamma)$	Bandwidth improvement factor	Defined by Hansen [4, 5, 13] (Bode, Fano and Wheeler never quantified this Hansen-defined factor) $BWIF_n = \delta_1/\delta_n = B_n/B_1$ (Other $BWIF$s can be defined. In [3, 7] Lopez defined $BWIF = B_n/B_{(n-1)}$, which is helpful in determining the percent bandwidth increase associated with increasing the network complexity by one tuning section. It is better suited for quantifying the law of diminishing returns with increasing network complexity.)

Table 2. A comparison of Hansen's δ_n to Lopez's $1/QB_n$ for $VSWR = 2$ ($\Gamma = 1/3$).

n	Hansen's [1,4,5] δ_n	Lopez [3]		
		$1/QB_n$	a_n	b_n
1	1.33333	1.33333	1	1
2	0.57735	0.57735	2	1
3	0.46627	0.46606	2.413	0.678
4	0.42416	0.42386	2.628	0.474
5	0.40264	0.40247	2.755	0.347
∞	0.34970	0.34970	π	0

should be replaced by

$$QB_n = \frac{1}{b_n \sinh\left[\frac{1}{a_n}\ln\left(\frac{1}{\Gamma}\right)\right] + \frac{1-b_n}{a_n}\ln\left(\frac{1}{\Gamma}\right)}, \quad (5)$$

Lopez's [3] Equation (7). (Note that a_n and b_n are not the same as Fano's a and b. Fano's a and b depend on Γ. Lopez's a_n and b_n are given in Table 2.)

In Table 2, the results of Equation (5) are compared directly to Hansen's δ values, presented in his Table 1a for $VSWR = 2$, showing close agreement.

In Hansen's Table 2 [1], the results for the column labeled "Lopez [3]" were not in agreement with the other results. Hansen mistakenly listed these results as *BWIF*. They actually were for *QB* or $1/\delta$. The inverses of these results are in close agreement with the δ results presented in Hansen's [1] Table 1. Also, in his Table 2, the column labeled "Fano" should be changed to "Hansen [1]." Fano's equations do not directly quantify the Hansen-defined *BWIF*.

"Lopez coefficients are different from the Fano coefficients. The former were determined to minimize the BWIF *error over a range of* Γ*."*

The second sentence is not correct: Equation (5) quantifies QB_n for all values of Γ. Lopez's a_n and b_n coefficients do not depend on Γ, and are exact for $n = 1$, 2, and ∞. The other coefficients, up to $n = 8$, were determined such that the accuracy in quantifying QB_n was better than 0.1%.

"In an earlier paper, Lopez [4] recommended a different formula:"

This statement is not correct. In a *later* rebuttal paper, Lopez [8, Equation (4)], for the purpose of comparison to Hansen's results, presented an equation for the Hansen-defined bandwidth improvement factor:

$$BWIF = \frac{B_n}{B_1} = \frac{\sinh\left[\ln\left(\frac{1}{\Gamma}\right)\right]}{b_n \sinh\left[\frac{1}{a_n}\ln\left(\frac{1}{\Gamma}\right)\right] + \frac{1-b_n}{a_n}\ln\left(\frac{1}{\Gamma}\right)}, \quad (6)$$

$$n = 2, 3, 4, \ldots, \infty.$$

Lopez used Equation (5) [3] to derive Equation (6) [8].

4. Historical Errors

"Wheeler [5], in a monograph on matching, included data on BWIF. *These data were based on the Bode limit:*

$$BWIF = \frac{\pi}{\ln\frac{1}{\Gamma}}. \quad (7)$$

(Hansen's [1] incorrect Equation (3))

The Wheeler monograph does not include data on the Hansen-defined *BWIF*. Wheeler's fundamental work on single tuning ($n = 1$) and double tuning ($n = 2$) impedance matching was not based on the Bode limit. It was not derived from Bode's nor Fano's work. Wheeler developed his results independently, using the reflection (Smith) chart as his analysis tool [10]. Reference [9] for this article includes a hyperlink for the download of the Wheeler monograph.

Equation (7) (Hansen's [1] incorrect Equation (3)), which he mistakenly attributed to Bode, should be replaced by

$$QB_\infty = \frac{\pi}{\ln\frac{1}{\Gamma}}, \quad (8)$$

again, because $BWIF \neq QB_n$ (see also Appendix 2).

"A careful reading of Bode [7] reveals that he assumed that 'the reflection coefficient is constant in the prescribed range.' His result, Equation (3) above, is thus severely limited."

The logic in this statement is difficult to understand. Bode assumed constant reflection, which corresponds exactly to the case of $n = \infty$, as clearly stated by Fano [8] (see Appendix 3). In the limit of an infinite number of tuned circuits, the reflection magnitude within the frequency band approaches the constant value

$$\Gamma = e^{-\frac{\pi}{QB}}.$$

Outside the band, it approaches

$$\Gamma = 1.$$

This is a reasonable result, and is not severely limited.

5. Bode Corrected

"Bode Corrected" should be changed to "Correct Hansen *BWIF* for Fano-Bode Case." Hansen presents correct formulas for the Hansen-defined *BWIF* for the Fano-Bode case, $n=\infty$:

$$BWIF_{\infty}=\frac{\delta_1}{\delta_{\infty}}=\frac{B_{\infty}}{B_1}=\frac{QB_{\infty}}{QB_1}=\frac{\dfrac{\pi}{\ln\left(\dfrac{1}{\Gamma}\right)}}{\dfrac{1}{\sinh\left(\ln\dfrac{1}{\Gamma}\right)}}=\frac{\pi\sinh\left(\ln\dfrac{1}{\Gamma}\right)}{\ln\dfrac{1}{\Gamma}}. \quad (9)$$

This result agrees with Equation (6).

6. Conclusion

"The limitations of the simple Bode criterion are now obvious, and the results [3, 5, 6, 7] (Lopez, Wheeler, Tanner, Bode) should no longer be used."

These statements by Dr. Hansen are unsubstantiated and are not true. The results of [7, 9, 6, 11] (Lopez, Wheeler, Tanner, Bode) should be used, and Lopez's formulas and graphs [3, 7, 8] are correct. It is hoped that this rebuttal sets the record straight.

7. Appendix 1: Substantiation that $QB=\dfrac{2\omega_c}{A_1^{\infty}}$

The following material in quotation marks in this appendix is taken directly from Fano [2, pp. 139, 140]. The items in square brackets have been added by Lopez.

"A very simple and important type of matching problem is presented by the case of a load impedance consisting of a resistance in series with an inductance."

"...the pass band desired in most of these problems extends from zero frequency to some cut-off frequency ω_c."

$$\left[\omega_c=\omega_{High}-\omega_{Low}\right]$$

"Let L_1 be the value of the inductance normalized with respect to the series resistance, that is divided by it."

$$\left[L_1=L/R\right]$$

"The coefficient A_1^{∞} is, by definition,

$$A_1^{\infty}=\left[\frac{d}{d\dfrac{1}{\lambda}}\left(\ln\frac{2+\lambda L_1}{\lambda L_1}\right)\right]_{\frac{1}{\lambda}=0}=\frac{2}{L_1}. \quad (31)\text{"}$$

From the above it follows that

$$\frac{2\omega_c}{A_1^{\infty}}=\frac{2\left(\omega_{High}-\omega_{Low}\right)}{\dfrac{2}{L_1}}=\frac{\omega_0 L}{R}\frac{\left(\omega_{High}-\omega_{Low}\right)}{\omega_0}=QB,$$

$$\omega_0=\sqrt{\omega_{High}\omega_{Low}},$$

$$QB=\frac{2\omega_c}{A_1^{\infty}}.$$

8. Appendix 2: Substantiation that the Bode Limit is Given by $QB_{\infty}=\dfrac{\pi}{\ln\dfrac{1}{\Gamma}}$

The following material in quotation marks in this appendix is taken directly from Bode [11, p. 367]:

"If ω_1 and ω_2 represent the edges of the prescribed band, this allows (16-13) to be written as

$$\int_{\omega_1}^{\omega_2}\log\left|\frac{1}{\rho}\right|d\omega\le\frac{\pi}{CR_1}, \quad (16\text{-}15)$$

where the equality sign obtains in the limiting case when $\sum a_j=0$ and R [the real part of the input impedance] is negligible below ω_1 and above ω_2.

"The simplest example of (16-15) is found, of course, when the reflection coefficient is constant in the prescribed range. We then have

$$\log\left|\frac{1}{\rho}\right|\le\frac{\pi}{\left(\omega_2-\omega_1\right)CR_1} \quad (16\text{-}16)\text{"}$$

From the above, we obtain the equation for the Bode Limit:

$$\ln\frac{1}{\Gamma}=\frac{\pi}{\dfrac{\omega_0 C}{G_1}\dfrac{\omega_2-\omega_1}{\omega_0}}=\frac{\pi}{QB_{\infty}},$$

$$\omega_0=\sqrt{\omega_1\omega_2},$$

$$QB_{\infty}=\frac{\pi}{\ln\dfrac{1}{\Gamma}}.$$

9. Appendix 3: The Fano-Bode Limit

The following material in quotation marks in this appendix is taken from Fano [2, p. 71]:

"It is clear that the best possible utilization of this area is obtained when $\ln 1/|\rho_1|$ is kept constant over the desired frequency

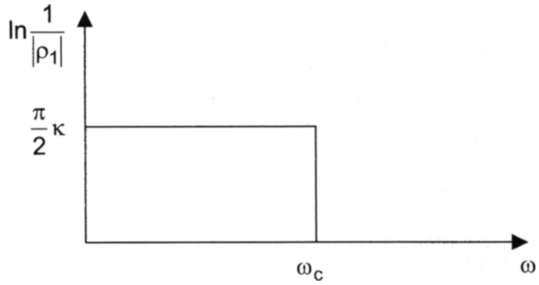

Fig. 6. Optimum frequency response

Figure 1. Figure 6 from Fano [2].

band and is made equal to zero over the rest of the frequency spectrum. This situation is illustrated for the low-pass case in Fig. 6 [reproduced herein as Figure 1].

"If w is the desired bandwidth ($w = \omega_c$ in Fig. 6) the best possible tolerance is given by

$$\left[\ln\frac{1}{|\rho_1|}\right]_{\max} = \frac{\pi}{2w}A_1^{\infty} \tag{24}$$

"This theoretical limitation was first found by Bode, as pointed out above. In fact, when the load consists of a parallel RC combination, the coefficient A_1^{∞} becomes equal to $2/RC$."

From the above, it is clear that Fano understood that in the limit of $n = \infty$, his result was identical to that of Bode. Consequently, this limiting case is referred to as the Fano-Bode limit.

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