Calibration of LAAS Reference Antennas

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BIOGRAPHY

Alfred R. Lopez is a Life Fellow of the IEEE. He received a BEE from Manhattan College in 1958 and an MSEE from the Polytechnic Institute of Brooklyn in 1963. He is a Hazeltine Fellow at BAE SYSTEMS Advanced Systems. He started his career at Wheeler Laboratories in 1958 as an antenna design specialist. He has made contributions to the theory and practice of electronic scanned antennas. From 1969 to 1990 he was involved with the development of the Microwave Landing System. He has published extensively in IEEE publications, has been issued 33 US Patents, and has received several IEEE Awards; one being the 1988 IEEE Antennas and Propagation Society's Harold A. Wheeler Award.

ABSTRACT

The Differential GPS, DGPS, Local Area Augmentation System, LAAS, utilizes reference antennas and receivers to measure the time of arrival of GPS signals at precisely surveyed points. These measurements are then used to broadcast differential corrections to approaching aircraft. A common misconception is to assume that the antenna phase center is the precise point whose position is being measured. The antenna phase center (or equivalent carrier phase-delay center) is a well defined concept: "The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extents into the far-field, the phase of a given field component over the surface of the radiation sphere is essentially constant, at least over that portion of the surface where the radiation is significant," (IEEE definition). The antenna phase center is defined at one frequency, the carrier frequency. For GPS reference antennas, a new antenna concept, the antenna group phase center (or equivalent code phase center) should be defined. Thus, the antenna has two phase centers, the carrier phase center and the code phase center. These phase centers are not necessarily points and the two phase-delay centers may or may not have the same characteristics. Typically, they do not have the same characteristics.

This paper introduces the concepts of code-phase delay and carrier-phase delay as related to the calibration of LAAS reference antennas. It describes the characteristics of one candidate type of reference antenna for LAAS. It discusses the results of some recent field measurements of antenna code-phase-delay minus carrier-phase-delay. It also discusses the measurements of code and carrier phase delays, which may be used to calibrate GPS reference antennas.

INTRODUCTION

The Local Area Augmentation System is a local differential GPS, DGPS, system that is being developed by the Federal Aviation Administration and the aviation industry to support high-precision aircraft approach procedures. It consists of a small collection of high-quality GPS receivers and antennas at known, surveyed locations on an airport property. LAAS determines range corrections that are broadcast to approaching aircraft. The airborne receiver uses these measurements to correct it's own measurements to achieve sub-meter accuracy.

The reference antennas have stringent accuracy requirements; the error directly attributed to the antennas should not exceed a few centimeters. The antennas, like other system components, have a transmission-line type delay that is included with the delays of the other components in the basic calibration of the system. The special errors associated with the antennas that are of concern in this paper are the errors that are angle dependent. These errors are related to the inherent variation with angle of the antenna "carrier phase center" and the antenna "code (group) phase center." The antenna phase center is a well defined concept [1], "The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extents into the far-field, the phase of a given field component over the surface of the radiation sphere is essentially constant, at least over that portion of the surface where the radiation is significant," (IEEE definition). The writer is not aware of a definition for the antenna "group (code) phase center" and is proposing that the IEEE definition of phase center be used to define code phase center by replacing the words, "the phase of a given field component," by "the code phase of a given field component." Before defining code phase center, the code phase must be defined. The antenna carrier-phase and code-phase centers are not necessarily the same point.

For DGPS the observables of interest are the code phase delay and the carrier phase delay. The code delay is equal to the signal modulation or group delay, and is equal to the rate of change of phase with respect to angular frequency (dφ/dω). The antenna code delay should be used to measure the code pseudorange correction. The carrier delay is defined at the carrier frequency, and is equal to the total phase divided by the angular frequency (-φ/ω). For LAAS calibration purposes, measurement of the code delay variation, over the antenna coverage region, is a requirement.

An outline of the paper is presented below:

- Overview of Calibration Process
- The Antenna Phase Centers
- Rudimentary Antenna Example
- BAE SYSTEMS Model ARL-1500 Antenna
- Antenna Calibration Methodology
- Summary

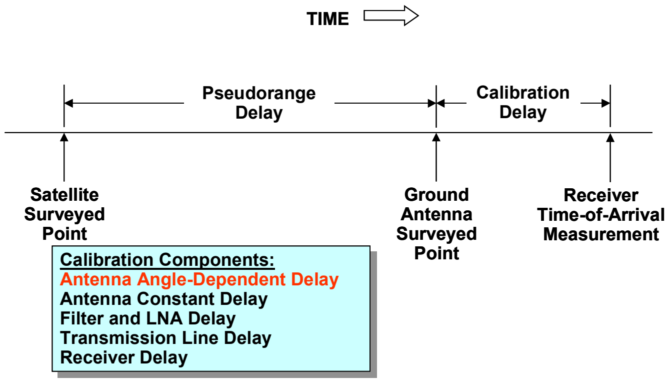


Figure 1. Pseudorange Measurement

OVERVIEW OF CALIBRATION PROCESS

Some basic elements for a pseudorange measurement are shown in Figure 1. It is assumed that the time-of-departure of the code epoch from the surveyed point at the satellite is known. The time-of-arrival of the code epoch at the ground antenna surveyed point can not be measured directly because of intervening components that add delay to the measurement. These components, constant and angle-dependent, are indicated in Figure 1. The magnitude of the code (group) delay for the constant components can be determined and subtracted from the time-of-arrival as measured at a point within the receiver.

The antenna angle-dependent code-delay component is characteristic of the antenna in a manner analogous to the typical antenna angle-dependent amplitude and phase characteristics. An antenna range measurement is required to determine the code delay (phase) variation with angle. This measurement can be used to reduce the pseudorange correction error associated with the antenna angle-dependent code-delay component.

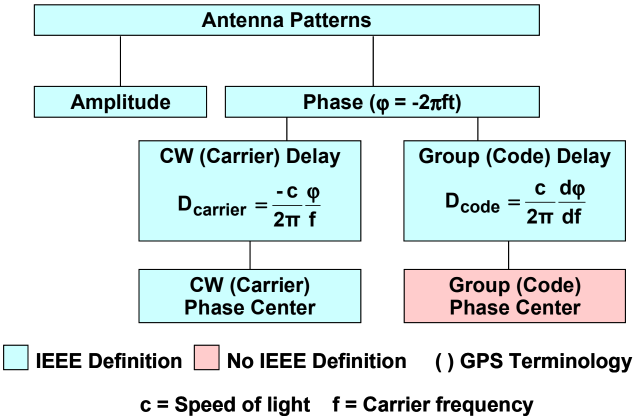


Figure 2. Antenna Angle Dependence

THE ANTENNA PHASE CENTERS

The angle-dependent variations of an antenna are described in terms of antenna patterns, the spatial distribution of a quantity that characterizes the electromagnetic field generated by an antenna [1]. As shown in Figure 2, antenna patterns can be either amplitude or phase patterns. Phase patterns are divided into two subclasses, CW delay and group delay. The IEEE defines the concept of antenna phase center with respect to CW radiation [1] where phase center is synonymous with carrier phase center. For DGPS there is a need to define another fundamental antenna characteristic, the antenna code (group) phase center. To define the code phase center it is first necessary to define code phase. Proposed definitions for code phase and code phase center are presented in Figure 3. Some basic characteristics associated with the carrier phase center and the code phase center are presented in Figure 4.

In the evolution of DGPS, the common perception was that of the antenna phase center. This concept should now be expanded to the notion of the antenna phase centers, the carrier phase center and the code phase center.

Code Phase	**For GPS, the ratio of the code delay, in units of time, to the code repetition period.**
Code Phase Center	**The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extends into the far-field, the code phase of a given field component over the surface of the radiation sphere is essentially constant, at least over that portion of the surface where the radiation is significant. Note: Some antennas do not have a unique code phase center.** **(This proposed code phase center definition is identical to the IEEE definition for phase center with the word "phase" replaced by "code phase.")**

Figure 3. Proposed Definition – GPS Code Phase Center

Phase centers may or may not be a point.

Phase centers may or may not be the same.

If one phase center is a point, then, the carrier and code phase centers are the same point.

Typically, there is significant variations and significant difference in the variations of the carrier and code phase centers

The highest possible DGPS accuracy is achieved by calibration of the reference antenna phase center<u>s</u>.

Figure 4. Phase Center Characteristics

Some recent measurements at the FAA William J. Hughes Technical Center have shown a systematic relationship between the variation of the mean-value of code-minus-carrier pseudorange error and the antenna amplitude pattern [3]. It is believed by the writer that this relationship can be explained by a comparison of the amplitude variation to the inherent variation in the difference between the code and carrier phase centers. This will be discussed in more detail in the following sections.

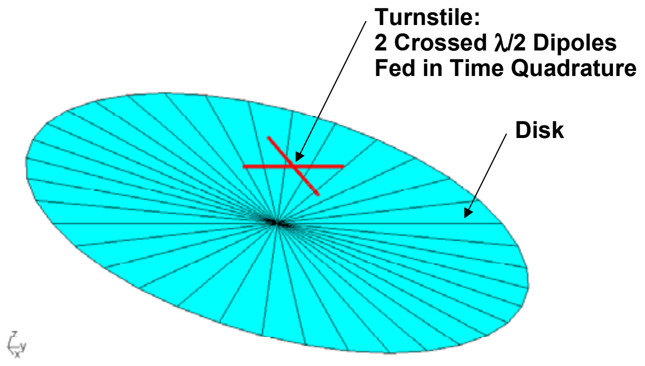


Figure 5. Rudimentary Antenna Example – Turnstile-Disk Antenna (λ = wavelength)

RUDIMENTARY ANTENNA EXAMPLE

Analysis of a rudimentary antenna will show that an angle-dependent code phase delay is characteristic of practically all ground reference antennas. An example of a simple theoretical GPS antenna is shown in Figure 5, the antenna consists of a turnstile element [2] (two crossed half-wave dipoles, fed in phase quadrature to produce right-hand circular polarization) located a quarter wavelength above a 0.5m diameter metal disk. The following analysis is performed at a frequency of 1575 MHz.

We start by considering the characteristics of the turnstile antennas without the reflector. For this case the total radiated power gain, dBi,, and the gain with respect to right-hand circular polarization, dBiRC, are shown in Figures 6(a) and 6(b). It is noted that the total radiation, dBi, is almost isotropic. The radiation with respect to a right-hand circularly polarized source, dBiRC, has a null in the nadir direction, because left-hand circular polarization is characteristic of the antenna in this direction. The computed carrier phase and code phase are identical, as shown in Figure 6(c). Both the carrier phase center and the code phase center are located at the center of the turnstile element.

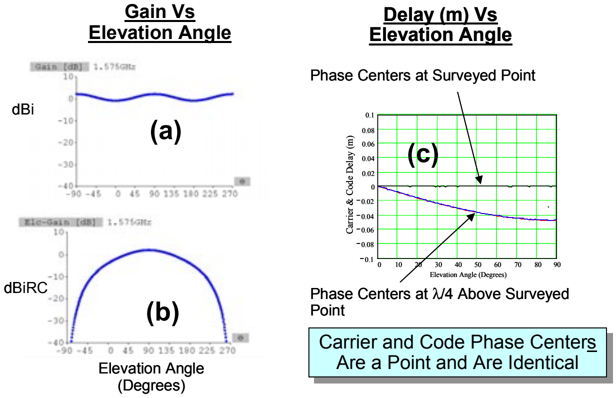


Figure 6. Characteristic of Turnstile Antenna (No Disk)

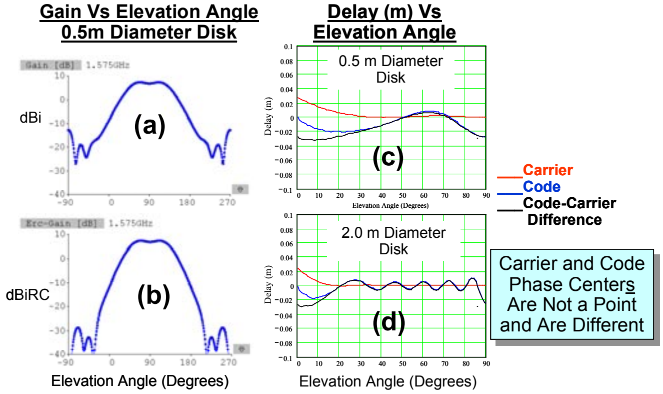


Figure 7. Characteristics of Turnstile-Disk Antenna

Radiation patterns for the turnstile-disk antenna are shown in Figures 7(a) and 7(b). The disk creates the pattern cutoff near the horizon (0° and 180° elevation angles). The phase characteristics are also shown in Figure 7(c). The antenna reference point is at the center of the disk.

Of special interest is the difference between the code and carrier phase delays. Above 30° of elevation angle, the carrier phase delay is nearly constant, which is characteristic of the antenna if the disk diameter is large. The disk creates an image of the turnstile antenna and the resulting 2-element array has a carrier phase center at the center of the disk. It is noted that the code phase delay has significant variation over the complete range of elevation angle. This variation is attributed to diffraction by the rim of the disk. The diffraction component acts like multipath with a delay that is approximately equal to:

$$D_{Dif} = R(1-\cos(\theta))$$

Where

R = Disk radius

θ = Elevation angle

This multipath-like component creates a variation of the code delay with elevation angle. This is more clearly illustrated in Figure 7(d), which shows the variation for the case of a disk with a 2m diameter. One can observe the increased number of cycles in the code phase variation associated with diffraction from the rim of a larger diameter disk. Appendix A provides an independent theoretical verification of the code-carrier difference variation of rudimentary antennas.

The key result is the observation that the addition of a reflector to the turnstile antenna, which has identical code and carrier phase characteristics, creates a significant difference in the code and carrier phase characteristics. It is expected that most GPS LAAS reference antennas have code and phase characteristics that differ significantly. The code-phase variation should be used for the calibration of DGPS reference antennas. The carrier-phase variation should be used for the calibration of antenna systems that only utilize the carrier phase in their operation. The key electrical specifications for a centimeter-accuracy DGPS reference antenna are presented below.

- Gain (dBiRC) over coverage volume (upper hemisphere)
- Up-down gain ratio (dB) (ratio of total radiated power, provides suppression of ground multipath)
- Antenna reference points (x-y-z-coordinates of the average carrier phase center and the average code phase center)
- Carrier phase center variation (mm) over coverage volume (calibration data for measurements that utilize carrier phase)
- Code phase center variation (cm) over coverage volume (calibration data for measurements that utilize the code epoch)

BAE SYSTEMS MODEL ARL-1500 ANTENNA

An antenna was conceived in 1995 [4], which was intended for application as a GPS ground reference antenna. The key features of this antenna are:

- Single port coverage of upper hemisphere with right hand circular polarization
- Sharp pattern cutoff at horizon for acquisition of satellites at low elevation angles
- High up/down gain ratio for suppression of ground multipath error
- Operation at L1 and L2 frequencies

Several prototypes of this antenna have been fabricated [5] and tested at various facilities. An initial evaluation indicates that this antenna shows promise of satisfying the requirements for DGPS ground reference antenna systems. Key to its performance is the reduction of angle-dependent code delay error by means of calibration.

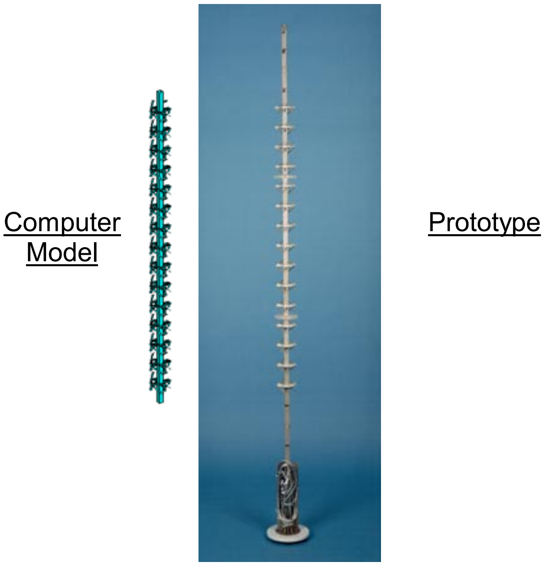


Figure 8. BAE SYSTEMS Model ARL-1500 Antenna

An FAA William J. Hughes Technical Center report [3] describes a trend in the mean of the code-minus-carrier measurement of the pseudorange error that is related to the antenna gain (amplitude) pattern for the Multipath Limiting Antenna [9]. The report indicates that the trend would impact error characterization. It speculates that the trend is related to amplitude and that perhaps some parameter, such as AGC, could remove the trend. The writer believes that the trend is not directly related to amplitude but, rather, it is related to a fundamental difference between the code-delay and carrier-delay antenna patterns and, that the trend can be removed by calibration of the antenna.

The computer model (see Figure 8) that was used for the initial design of the Model ARL-1500 antenna was used to compute the code-carrier difference delay pattern. Antenna phase patterns were computed at two frequencies separated by 10 MHz. The code-carrier difference delay pattern is then computed using the formulas given in Figure 2. This pattern was then compared to the computed amplitude pattern. Shown in Figure 9 are the computed gain patterns; the total gain, dBi, and the gain with respect to a right-hand circularly polarized source, dBiRC. It is noted that over the upper hemisphere (0° to 180° elevation angles) the dBi and dBiRC responses are nearly identical. This indicates that the antenna has essentially right hand circular polarization over the upper hemisphere.

The dBiRC gain pattern was converted to carrier-to-noise-density ratio pattern and is presented in Figure 10. Figure 10 also shows the code, carrier and code-carrier difference delay patterns with elevation angle. One can see a definite relationship between the amplitude (gain) variation and the code-carrier difference variation. It is this type of relationship that was observed in the FAA William J. Hughes Technical Center measurements [3]. If the antenna code-delay variation with elevation angle can be determined then it is possible to eliminate this pseudorange correction error component by a calibration process.

It is noted that the code delay (phase) pattern is a fundamental antenna characteristic, as is the antenna amplitude pattern and the carrier delay (phase) pattern. They exhibit similar traits.

It is also noted that the use of “B Values” [8] to assess the performance of ground reference antennas could result in significant error and degradation of system integrity. The “B Values” is a comparison of pseudorange corrections from several reference antennas, and is a measure of the integrity of the LAAS Ground Facility, LGF, in a multipath environment. A within-specification difference in the pseudorange corrections amongst the reference antennas is intended to indicate high integrity. If the accuracy of a particular antenna type is determined by a “B Value” assessment of a group of antennas, then, if all antennas have identical and out-of-tolerance variation of code delay with antenna angle and the multipath environment is benign, the antennas would be judged to have acceptable performance. This is because at any antenna angle they would all have the same pseudorange correction error associated with the code delay variation with antenna angle. For this case the LGF would lack integrity because it is broadcasting within specification conditions when in fact the pseudorange corrections have out-of-tolerance errors. What is needed is an independent assessment of each antenna to ensure that the angle-dependent error is within specification. A calibration process may be needed to satisfy the requirements.

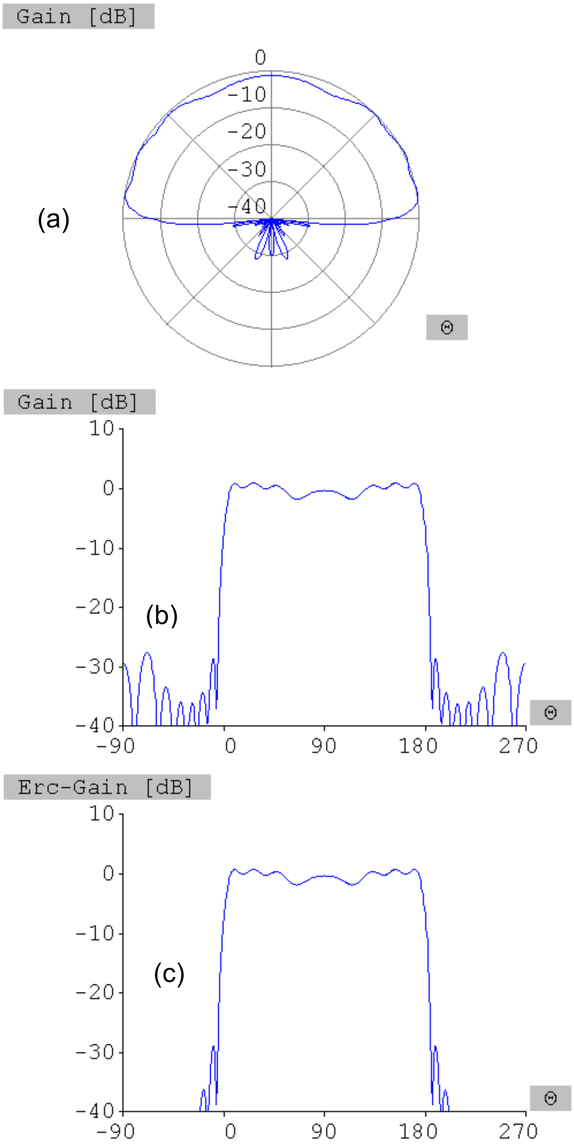


Figure 9. BAE SYSTEMS Model ARL-1500 Antenna (a) total gain, dBi, polar plot (b) total gain, dBi (c) gain with respect to right-hand circular polarization, dBiRC

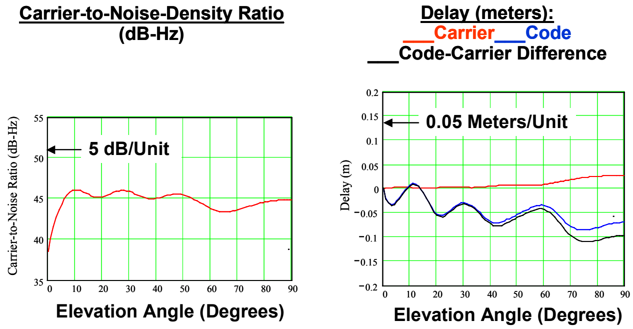


Figure 10. BAE SYSTEMS Model ARL-1500 Antenna – Computer Simulations – Amplitude, Carrier Delay, and Code Delay Patterns

ANTENNA CALIBRATION METHODOLOGY

Measuring the code delay variation over angle, and using this data for calibration of the antenna, will reduce the antenna angle-dependent error associated with the code delay variation. To minimize the magnitude of the code delay variation over the coverage volume the physical reference point (surveyed point) should be at or near the median value of the code phase center evaluated over the desired coverage volume.

The measurement of code delay variation can be performed at an antenna range or on a site using the satellite constellation. In either case, the measurement is somewhat difficult to perform. A high-quality antenna range is required for measuring the antenna code delay pattern. An outline of the two alternatives for code-delay measurements is presented below.

- Conventional Antenna Test Range
  - Very high quality (ideally –50 dB reflection level)
  - Measurements – Gain pattern, up-down ratio pattern, carrier delay pattern, and code delay pattern
- Site Installation Using GPS Constellation As Source Radiation
  - Far-field pattern (20,000Km)
  - Satellite provide near constant illumination at ground level
  - Measurements – Carrier-to-noise-density ratio pattern, code-delay minus carrier-delay pattern, carrier delay pattern
  - Azimuth angle variation
    - Three 24 hour periods with antenna rotated 120° each 24 hour period
    - One 24 hour period with several 360° continuous antenna rotations in 24 hour period

Antenna Range Measurement

The specification for one type of antenna range, suitable for the measurement of code delay, is given in Table 1.

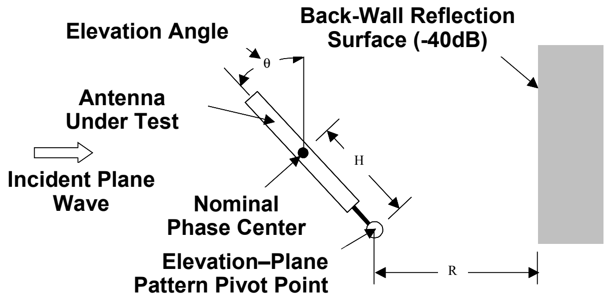


Figure 11. Test zone geometry for tapered anechoic chamber

Table 1. Antenna Test Range Specifications

Type range	Tapered anechoic chamber
Axial length	30m
Test zone dimensions	6m height 6m width 6m length
Back wall reflection factor	< -40 dB
Frequency range	1565 to 1585 MHz 1217 to 1237 MHz
Frequency steps	2.0 MHz
Antenna positions	Elevation, 0° to 90° in 0.2° steps Azimuth, 0° to 360° in 2° steps
Code phase delay error (rms)	< 0.01m

To evaluate the range error the antenna is assumed to have a point carrier-phase center and a point code-phase center that are coincident as shown in Figure 11.

The back wall reflection is the dominant error component for a tapered anechoic chamber. The antenna-range carrier phase error is given by [11]:

$$\varphi(f,\theta)=\sin^{-1}\left[\frac{-\rho_{ar}\sin\left[\frac{4\pi f}{c}(H\sin(\theta)+R)\right]}{\sqrt{1+\rho_{ar}^{2}+2\rho_{ar}\cos\left[\frac{4\pi f}{c}(H\sin(\theta)+R)\right]}}\right]$$

Where

ρ_{ar} = Antenna-range reflection factor
= $\rho_{ud}\,\rho$ = 0.0018 (-55 dB)
ρ_{ud} = Antenna up/down ratio factor
= 0.18 (-15 dB)
ρ = Back wall reflection factor = 0.01 (-40 dB)

The equations presented in Figure 2 are used to compute the carrier delay and the code delay.

The range error variation with elevation angle is shown in Figure 12. It is noted that the standard deviation for the range code-delay error is less than 0.01meters, which is considered to be suitable for the measurement of the code delay pattern.

Site Measurement Using Satellite Constellation

The satellite constellation and antenna site can be visualized as the largest antenna test range ever conceived. It has a far-field distance of approximately 20,000Km and the signal level at the test site is nearly constant for all satellites independent of their position in space., Amplitude, carrier delay, and code delay, antenna patterns are measured by rotating the reference antenna about its vertical axis and recording data during a test period (one to several days). The antenna rotation can be continuous or stepped. Each data point has an azimuth angle and elevation angle tag. The data points are sorted

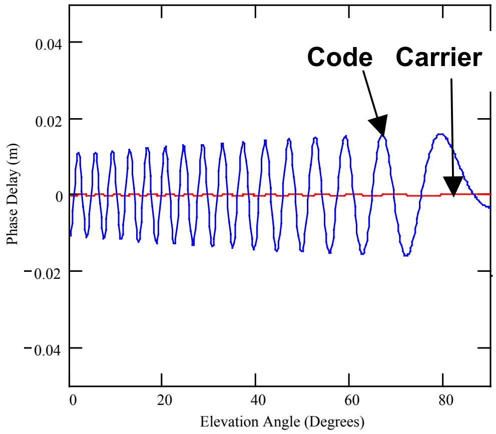


Figure 12. Antenna range delay error versus elevation angle, H = 1.5m, R = 3m, f = 1575 MHz, and $\rho_{ar} = 0.0018$

into azimuth and elevation bins and processed to generate the desired antenna patterns. Rotation about the vertical axis provides the means for reducing site multipath effects by averaging over many multipath scenarios.

Code-minus-carrier measurements [6], using the satellite constellation, have been used to quantify the pseudorange correction errors; assuming that the carrier-delay errors are negligible. This type of measurement can be used to determine the error variations that are characteristic of the reference antenna.

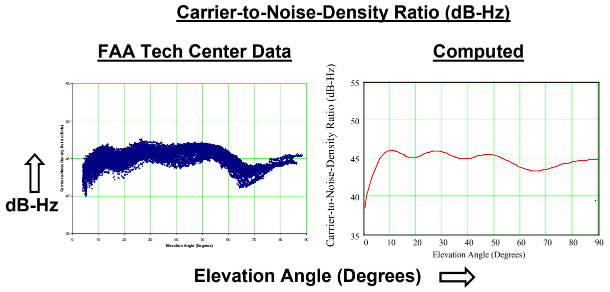


Figure 13. Amplitude versus elevation angle

The BAE SYSTEMS Model ARL-1500 antenna was measured at the FAA William J. Hughes Technical Center. The antenna (with conical ground plane) was installed at the LAAS Test Prototype (LTP) field site at one of the four established LTP antenna locations. GPS observables were collected using the Novatel Millennium GPS Receiver and a laptop computer. An Astech Z-XII connected to an Ashtech survey ground plane antenna was used to collect L1-L2 data for estimation of the ionospheric divergence. Data was collected in 24-hour periods to allow for observation of the full constellation at the site. The data samples have 100 seconds of carrier smoothing and a sample period of 200 seconds.

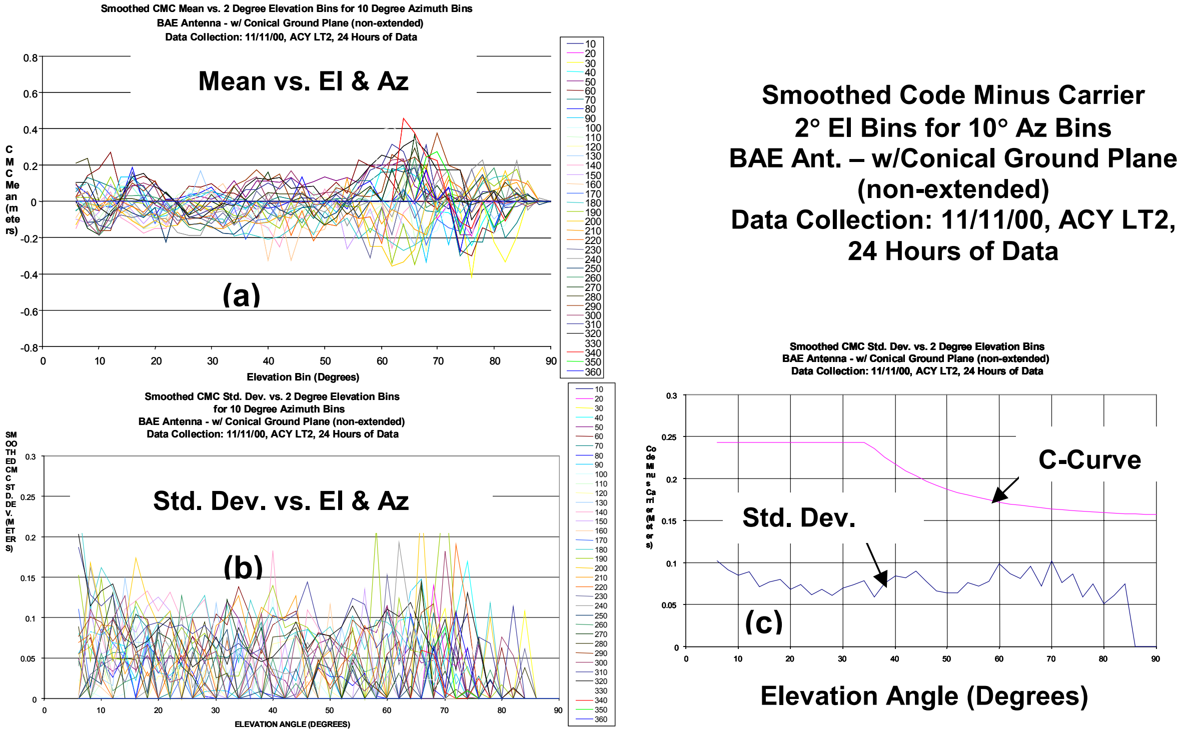


Figure 14. Model ARL-1500 antenna accuracy with elevation and azimuth calibration

Figure 13 shows a comparison of the Tech Center and computed amplitude versus elevation angle. The code-minus-carrier, CMC, mean versus 2°-elevation bins for 10°-azimuth bins is shown in Figure 14(a). This data set was further smoothed and interpolated to provide a complete set of calibration data for the complete range of azimuth and elevation angles. An analysis of the data (see Figure 15) indicates that that there is substantial systematic variation of the mean with azimuth. Because of the small size of the antenna in the horizontal plane, it is believed that the variation attributed to the antenna has a one-cycle variation in 360°. The indications are that an elevation-only calibration would not be satisfactory. A 2-dimensional (azimuth angle and elevation angle) calibration appears to be required for the BAE SYSTEMS Model ARL-1500 antenna.

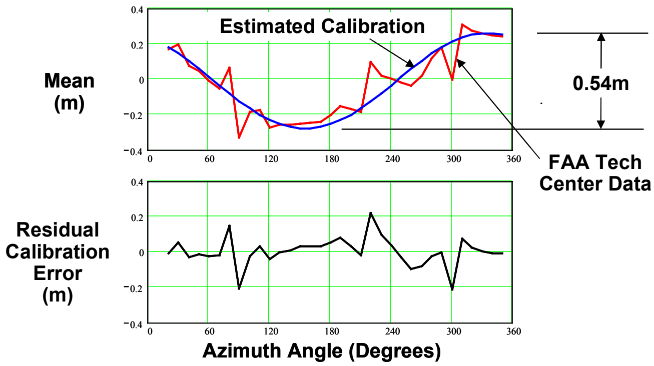


Figure 15. Mean pseudorange error versus azimuth angle at an elevation angle of 68° for BAE SYSTEMS Model ARL-1500 antenna

Figure 14(b) presents CMC standard deviation data for the same set of data presented in Figure 14(a). If it is assumed that the azimuth variation of the mean is characteristic of the antenna, then it can be removed by calibration. For this situation the standard deviation versus elevation angle can be computed for each 2°-elevation bin by computing the rms of the standard deviations for the 36 10°-azimuth bins. Figure 14(c) presents the results of this computation. Shown in Figure 14(c) is the so-called "C-Curve" LAAS accuracy requirement for one reference antenna [8].

A Model ARL-1500 antenna system accuracy budget is presented in Table 2. The budget includes a 0.08-meter allocation for a residual error associated with the calibration process. The budget indicates that the Model ARL-1500 antenna system could satisfy the LAAS Ground Facility accuracy requirement with some margin if the angle-dependent code-delay error calibration is successfully implemented.

Table 2. Model ARL-1500 Antenna System Accuracy Budget

	Sigma Pseudorange Correction Accuracy (Meters)
Noise & Multipath	**0.10**
Antenna with Calibration	**0.08**
Root Sum Square	**0.13**
Requirement: **El Angle < 35°** **El Angle = 90°** **(See C-Curve in Figure 14(c))**	 **≤ 0.24** **≤ 0.16**

SUMMARY

- The central theme of this paper is that the common perception of the antenna phase center should be expanded. It should be recognized and appreciated that there exists two phase-centers that have significance and relevance for DGPS. The usual phase (carrier phase) center is well defined. This paper proposes a definition for the code (group) phase center.
- Over the antenna coverage region, the average carrier phase center should be designated as the carrier phase center and the average code phase center should be designated as the code phase center.
- The antenna code-delay pattern can be measured at a very high-quality antenna range, or at a site using the satellite constellation. The site may be a convenient one or the actual operational LAAS site.
- Measurements of the antenna angle-dependent code delay can be used as calibration data that, in principle, reduces the antenna angle-dependent DGPS pseudorange correction error to zero.
- An elevation and azimuth calibrated BAE SYSTEMS Model ARL-1500 single-port L1-L2 antenna shows promise of satisfying the LAAS, WAAS, JPALS and CORS requirements.

ACKNOWLEGEMENTS

Special credit is given to Dave Lamb of the FAA William J. Hughes Technical Center; he was the first to observe a relationship between the code-minus-carrier delay characteristics and the reference antenna amplitude characteristics. The help provided by Dave Lamb, John Warburton and Mark Dickinson (all with the same FAA group) with the collection, processing, and interpretation of field data is greatly appreciated. At BAE SYSTEMS Advanced Systems, Edward Newman, Conrad Koch and Gary Nolan provided support, encouragement and help.

APPENDIX A: CODE-CARRIER DIFFERENCE VARIATION DEMONSTRATED BY SIMPLE DIFFRACTION ANALYSIS OF RUDIMENTARY 2-DIMENSIONAL ANTENNA

The radiation pattern for a 2-dimensional antenna consisting of a magnetic line source located at the center and directly above a perfectly conducting infinite strip, is relatively simple to compute using elementary diffraction theory [12] [13] [14]. Figure A2 present a Mathcad program that computes the code-carrier difference variation.

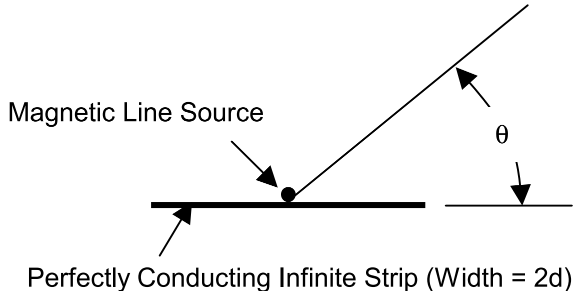
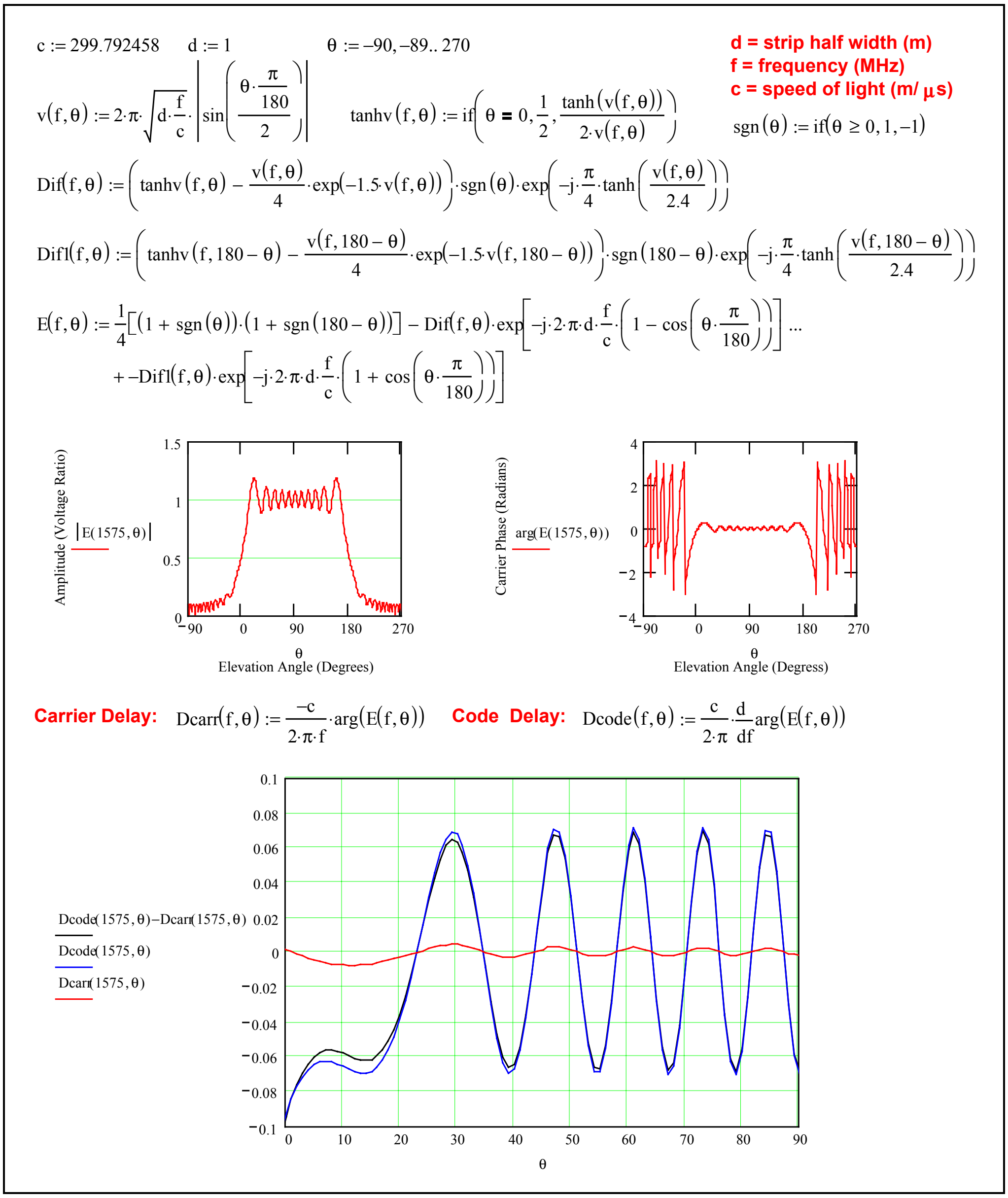


Figure A1. 2-D antenna geometry



$c := 299.792458 \qquad d := 1 \qquad \theta := -90, -89 .. 270$

d = strip half width (m)
f = frequency (MHz)
c = speed of light (m/ μs)

$$v(f,\theta) := 2\cdot\pi\cdot\sqrt{d\cdot\frac{f}{c}}\cdot\left|\sin\left(\frac{\theta\cdot\frac{\pi}{180}}{2}\right)\right| \qquad \mathrm{tanhv}(f,\theta) := \mathrm{if}\left(\theta = 0, \frac{1}{2}, \frac{\tanh(v(f,\theta))}{2\cdot v(f,\theta)}\right) \qquad \mathrm{sgn}(\theta) := \mathrm{if}(\theta \ge 0, 1, -1)$$

$$\mathrm{Dif}(f,\theta) := \left(\mathrm{tanhv}(f,\theta) - \frac{v(f,\theta)}{4}\cdot\exp(-1.5\cdot v(f,\theta))\right)\cdot\mathrm{sgn}(\theta)\cdot\exp\left(-j\cdot\frac{\pi}{4}\cdot\tanh\left(\frac{v(f,\theta)}{2.4}\right)\right)$$

$$\mathrm{Dif1}(f,\theta) := \left(\mathrm{tanhv}(f,180-\theta) - \frac{v(f,180-\theta)}{4}\cdot\exp(-1.5\cdot v(f,180-\theta))\right)\cdot\mathrm{sgn}(180-\theta)\cdot\exp\left(-j\cdot\frac{\pi}{4}\cdot\tanh\left(\frac{v(f,180-\theta)}{2.4}\right)\right)$$

$$E(f,\theta) := \frac{1}{4}\left[(1+\mathrm{sgn}(\theta))\cdot(1+\mathrm{sgn}(180-\theta))\right] - \mathrm{Dif}(f,\theta)\cdot\exp\left[-j\cdot 2\cdot\pi\cdot d\cdot\frac{f}{c}\cdot\left(1-\cos\left(\theta\cdot\frac{\pi}{180}\right)\right)\right] \ldots$$
$$+ -\mathrm{Dif1}(f,\theta)\cdot\exp\left[-j\cdot 2\cdot\pi\cdot d\cdot\frac{f}{c}\cdot\left(1+\cos\left(\theta\cdot\frac{\pi}{180}\right)\right)\right]$$

Carrier Delay: $\mathrm{Dcarr}(f,\theta) := \frac{-c}{2\cdot\pi\cdot f}\cdot\arg(E(f,\theta))$ **Code Delay:** $\mathrm{Dcode}(f,\theta) := \frac{c}{2\cdot\pi}\cdot\frac{d}{df}\arg(E(f,\theta))$

Figure A2. Mathcad program for computing code-carrier difference variation for rudimentary 2-D antenna; __carrier delay __code delay __code-carrier difference, strip width = 2 meters, frequency = 1575 MHz

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